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Fourier transform holographic lens in the telecentric readout configuration

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The problem of designing a Fourier transform holographic lens, which images a finite set of input plane wave fronts into a finite set of output spherical wave fronts, is considered. An optimal holographic Fourier transform lens with minimum mean-squared wave front error averaged over the set of input wave fronts is usually described with low values for spherical aberration, coma and astigmatism by permitting distortion. In this paper, a telecentric readout geometry on the image side with reducing distortion is presented.

1. Introduction

In the recent years, the Fourier transform holographic lens of a given focal length and different shape factors depending on the curvatures of the object and reference wave fronts, has been analysed [1]–[5]. To get the best solution of the design, a scheme of holographic lens recording was usually based on the off-axis geometry. By that means a holographic lens recorded with any wave fronts will introduce aberrations if illuminated with the wave front differing from one of the wave fronts used to produce the lens. Generally, a HOE (Holographic Optical Element) of a point source does not satisfy the requirements imposed by the Fourier transform lens realization. Therefore, attempts at designing the Fourier transform lens were carried on recording the non-spherical wave fronts which satisfy the Fourier transform requirements in an extended range of spatial frequencies. An optimal holographic Fourier transform lens was recorded with the aid of a computer generated hologram or was based on an analytic solution involving optimization by minimizing the output deviations [1], [3].

In this paper, an on-axis Fourier transform holographic lens produced by recording the two axial symmetrical wave fronts, is described. It can be shown that like a Fresnel zone hologram of an axis point source it is an attractive alternative to the off-axis holographic Fourier transform lens used so far, especially for the large field angles and small apertures.

2. Parallel-beam Fourier transform

In order to obtain a Fourier transform holographic lens with low aberrations over

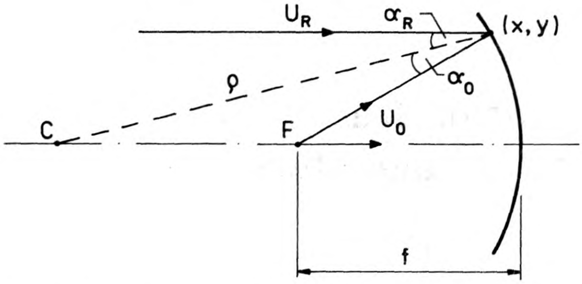

Fig. 1. Recording geometry of the HOE phase function on a spherical substrate

an extended range of spatial frequencies [6], [7], an on-axis holographic lens phase function on a spherical substrate was recorded. The recording arrangement is similar to that used in Gabor's experiment, and is illustrated in Fig. 1 [8]–[10]. The HOE phase function is then given by

$$\Phi_{\mathrm{H}}(x,y)=\frac{2\pi}{\lambda}\left[\sqrt{f^2+2(\rho-f)(\rho-\sqrt{\rho^2-x^2-y^2})}+\sqrt{\rho^2-x^2-y^2}-\rho\right] \tag{1}$$

where: f is the focal length (*i.e.*, distance between the point source emitting the spherical wave front and the vertex of spherical holographic lens), and ρ is the curvature radius of the spherical holographic lens. Analysing the above expression, we see that the phase function for a flat holographic lens can be obtained as a limit of the curved lens phase function if the curvature radius approaches infinity without bound. Then we have

$$\lim_{\rho\to\infty}\Phi_{\mathrm{H}}(x,y)=\frac{2\pi}{\lambda}\sqrt{f^2+x^2+y^2}.$$

The interference fringes of such a holographic grating form concentric circles, and the distance between adjacent rings which is inversely proportional to its spatial frequency, is given by

$$d=\lambda/(\sin\alpha_{\mathrm{O}}+\sin\alpha_{\mathrm{R}}). \tag{2}$$

Therefore, the minimal period of this HOE recorded on a spherical substrate has a form

$$d_{\min}=\frac{2\rho\lambda\sqrt{f^2+(\rho-f)(2\rho-\sqrt{4\rho^2-D_{\mathrm{H}}^2})}}{D_{\mathrm{H}}\left[\rho-f+\sqrt{f^2+(\rho-f)(2\rho-\sqrt{4\rho^2-D_{\mathrm{H}}^2})}\right]} \tag{3}$$

where D_{H} is the diameter of HOE. The functional relationship between minimal period and the curvature radius expressed by Eq. (3) is presented in Fig. 2, where:

$$d_{\min}^{0}=\frac{2\lambda f}{D_{\mathrm{H}}},\quad d_{\min}^{\max}=d_{\min}^{0}\sqrt{1+D_{\mathrm{H}}^2/4f^2}.$$

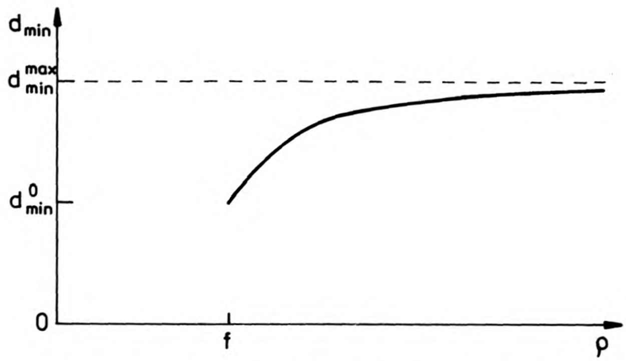

Fig. 2. Graphical representation of a functional relation between the minimal period of HOE and the curvature radius

The holographic lens of an axial point source formed by recording two axial symmetrical wave fronts, one of which is planar, as shown in Fig. 1, can satisfy the requirements of a Fourier transform lens covering an extended band of spatial frequencies. We see here an advantage in comparison with an off-axis geometry, if the input transparence is located in the front focal plane of holographic lens (at distance f from the vertex of HOE) and illuminated by collimated laser beam of wavelength λ. Let us remember that the case of optical Fourier transform realization by a lens is not an ordinary imaging process, where object points are imaged into image points. The object transparency inserted in the front focal plane of lens, illuminated by a plane wave propagating parallel to the axis, produces an angular spectrum of plane wave fronts, each of them propagating in different direction to the holographic lens and being focused to a corresponding point in the back focal plane. By this view, each of diffracted plane waves may be replaced by a coherent point source at infinity which corresponds to a particular spatial frequency. In such a situation, the conventional rules of image formation may be applied to images of corresponding object sources located at minus infinity, *i.e.*, to the conjugated planes: at infinity and in back focal plane. However, all the light rays emitted from the object point sources at infinity must penetrate the transparency in front focal plane of HOE, the aperture of which (dimensions of transparency) influences the image quality in the back focal plane.

Consider the geometry of ray tracing through a Fourier transform holographic lens recorded as shown in Fig. 1. According to this recording geometry, two unaberrated wave fronts in the image space are possible: the spherical one U_O which focuses to the focal point F, and the plane one U_R, the direction of which is parallel to the optical axis of the Fourier transform lens (Fig. 3). The first wave front (spherical) is transformed from plane wave propagating parallel to the axis, and the second one (plane wave front) is transformed from the spherical wave front emerging from the front focal point F located on axis in the input plane. The aperture stop in the Fourier transform configuration coincides always with the input transparency plane. This implies that the amount of light which reaches the image in the back

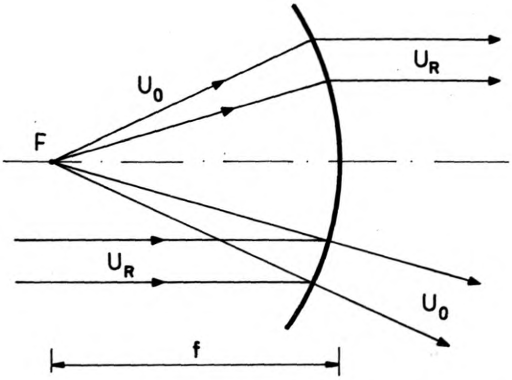

Fig. 3. Ray tracing of two unaberrated wave fronts through the curved Fourier transform holographic lens

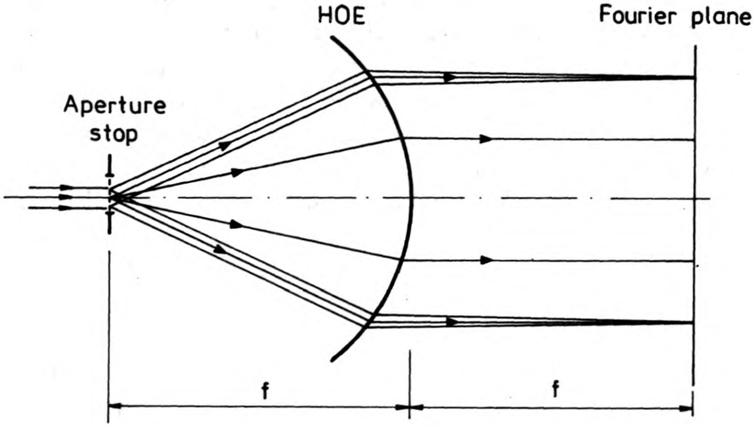

Fig. 4. Infinitesimal beams of diffracted rays at transparence focused in corresponding points of the output plane

focal plane depends on the dimensions of the input transparency. In each pencil of rays which proceeds to the holographic lens under the fixed diffraction angle α, one can find the chief ray passing through the centre of the aperture stop. Furthermore, the maximum diffraction angle α at the transparency, which is the field angle, determines the required diameter of the holographic lens, if this lens is to pass all spatial frequency components of the object transparency. If the centre of entrance pupil (aperture stop in this case) coincides with the point source which emanated spherical wave front during recording of this HOE, the chief rays of all beams defined by diffraction angle α at the input plane are aberration free after passing through the holographic lens and emerging parallel to its axis. Such a readout geometry can be very useful in measurements of the spatial frequencies of an angular spectrum of plane waves, and is said to be telecentric on the image side (Fig. 4). In this arrangement, the images will be located in the correct position of the output focal plane, because of reduction of the chief ray aberration, *i.e.*, distortion. When

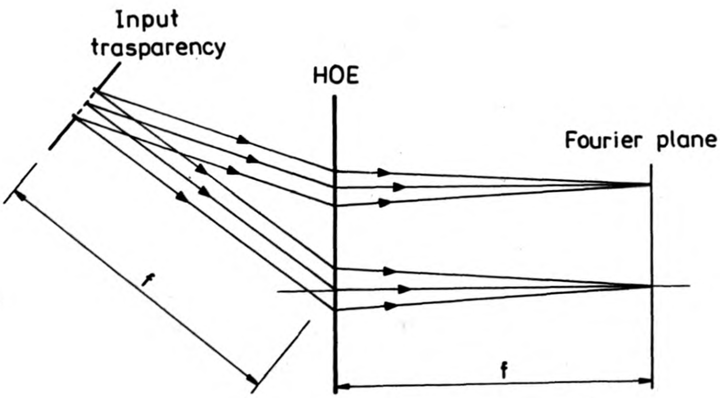

Fig. 5. Fourier transform off-axis HOE configuration

the area of the transparency is assumed to be small in comparison with the focal length, then infinitesimal pencils of diffracted rays travelling in different direction will be focused to the image points burdened with low aberrations. This tends to reduce the measurement error of the spatial frequencies caused by a slight defocusing of the output plane.

Till now, the Fourier transform holographic lenses were usually recorded on plane substrate with two wave fronts, *i.e.*, an axial spherical and an off-axis plane one, the latter being sometimes perturbed. During the Fourier transform realization, a tilted off-axis transparency is inserted in the input plane which is not parallel to the HOE plane, as shown in Fig. 5. In this case, the locations of image spots in the output focal plane are usually in wrong position, *i.e.*, distortion of the output is always present. We must remember that distortion depends very strongly on the position of the aperture stop. In the Fourier transform lens the aperture stop is identical with input transparency, being thus defined by the sizes of the latter. Therefore, it is all to the good, if the transparency is inserted in the front focal plane symmetrically with respect to the axis of the Fourier transform HOE (see Fig. 4); then the exit pupil is at infinity and the chief rays of all pencils representing the respective spatial frequencies of the input are parallel to the axis in the image space. In such a geometry, all points in the object plane placed at infinity will be depicted without or with reduced distortion in the conjugate output focal plane, because of fixed magnification, especially for small values of diffraction angles. Evidently this is equivalent to demanding that all the chief rays at varying obliquities must be free from spherical aberration in aperture pupils. This is the most important property of the proposed telecentric configuration.

3. Conclusions

The Fourier transform is, in general, optically performed by using the parallel beam setup in which the telecentric readout configuration is usually realized. Such an arrangement may be very useful in measurement location of image points representing the spatial frequencies of the investigated transparency, especially in the case of insignificant shift of the Fourier plane. The distance of the image point in Fourier plane from the axis can be determined by

$$x_F = f\tan\alpha.$$

If the chief ray follows Abbe's sine condition, the equivalent distance of the Fourier transform lens of equal focal length leads to

$$x'_F = f\sin\alpha.$$

Therefore, a holographic lens for Fourier transform can be defined by the condition that the image point at the position x_F is displaced by one resolvable spot size from the correct position. We are expecting that such a holographic element will provide for better performance particularly when recording an aspheric phase function.

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