ON THE VALIDITY OF THE TEST FOR ASYMMETRY IN RESIDUAL-BASED THRESHOLD COINTEGRATION MODELS

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The talk is the summary of the paper with the same title that resulted from the cooperation with Karsten Schweikert (University of Hohenheim), and was recently published (Schild/Schweikert (2019)).

1. Introduction

Informally, an *asymmetric price transmission* (APT) means that increases and decreases in input prices are transmitted to output prices at different speeds (e.g. ‘rockets and feathers’). Frequently, both prices are subject to unit-root non-stationarity, but co-integrate so that asymmetric cointegration (CI) techniques are required for the statistical analysis. Enders and Siklos (2001) suggest extending the Engle/Granger CI-procedure. They allow the cointegration errors to revert at different speeds from above and below, which is modelled by two-regime SETAR or MTAR, replacing the AR-model in Engle/Granger. To establish significance for an APT, Enders/Siklos suggest conducting two tests: first, test for CI (provide evidence for a co-integration), then test for APT (conduct a standard F-Test of equality of the two speeds). The test for APT (if conducted with conventional critical values) is valid only if the price series actually co-integrate. In effect, the second test will therefore be applied only if the first test is successful, i.e. if the data exhibit high significance for CI. When applied, an extension of the Enders-Siklos procedure is usually important in order to end up with the desired result (significance for an APT). The threshold involved in the SETAR or MTAR model to distinguish between

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the two regimes is not assumed to be given, but is also estimated from the data. This is usually done by a 'brute force search': estimate for many potential thresholds and take the one that provides the best fit.

In this talk the author demonstrates that this method can be heavily biased towards finding APT. One reason for this is the selection effect: passing the CI-test, the data presented to the APT-test are preselected in favour of asymmetry (intuitively: to obtain evidence for CI, the estimated reversion rates must be relatively large, which makes it more likely that they differ.) This selection bias is closely related to the small power of the CI-test and therefore it is primarily relevant when sample sizes or adjustment rates are small. This exists in both the basic and the extended version of the method.

However, the selection effect caused by requiring evidence for CI is not the whole story. In the simulations it was found that the bias (in the direction of falsely finding asymmetry) is strongly enlarged beyond the selection effect if the extended version is applied 'as usual' in the econometric practice ('as usual' means that the standard errors of the reversion rates do not account for the estimation of the threshold). As opposed to the selection effect, the asymmetry bias coming from an estimation of the threshold does not depend on the power of the CI-test. Most importantly, it appears to persist for large sample sizes. The basic problem with the extension is that the threshold is not identified under the null of the CI-test (but its estimation interacts with the estimation and standard errors of the two reversion speeds). It is not surprising that actually if any threshold is equally well fitting, the search for a threshold optimally fitting the data favours the estimation of 'significantly' different reversion rates.

A. Review of the Enders-Siklos procedure to test for APT

Let x_t and y_t denote the two prices, and assume that both have a unit-root instationarity. Assume that in the (potentially) co-integrating relationship

$$y_t \;=\; \beta_0 + \beta_1 x_t + z_t$$

the errors z_t satisfy

$$\Delta z_t \;=\; \underbrace{\varrho_+ \cdot I_t z_{t-1} + \varrho_- \cdot (1 - I_t)\, z_{t-1}}_{replaces:\ \ \varrho\, z_{t-1}\ \ in\ ADF\ and\ EG} \;+\; \varepsilon_t \quad \left[\,+\;\textstyle\sum_{j=1}^{k} \gamma_t \cdot \Delta z_{t-j}\right],$$

where ε_t represents a white noise process and

$$I_t := \begin{cases} 1 & z_{t-1} \geq \tau \\ 0 & z_{t-1} < \tau \end{cases} \quad (SETAR(\tau))$$

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$$\text{or } I_t := \begin{cases} 1 & \Delta z_{t-1} \geq \tau \\ 0 & \Delta z_{t-1} < \tau \end{cases} \quad (MTAR(\tau)).$$

The *threshold* τ is a constant which (for now) is assumed to be known (e.g. $\tau = 0$).

The coefficients $\varrho_\pm$ replace the unique ϱ in the ADF-Test and the Engle-Granger procedure. If negative, they represent the average *adjustment rates* at which the errors revert to 0 in the corresponding regime. If both are negative, z_t follows a stationary process, i.e. $x_t,\ y_t$ co-integrate, and if both are zero, z_t follows a unit-root-process, i.e. x_t, y_t do not co-integrate. (This is a simplified classification, sufficient for practical purposes).

The Enders-Siklos test for co-integration (CI-test) is analogous to the Engle-Granger two-step procedure: the first step is to estimate the regression $y_t = \beta_0 + \beta_1 x_t + z_t$ by OLS, the second step is to estimate the residual regression

$$\Delta z_t = \varrho_+ \cdot I_t z_{t-1} + \varrho_- \cdot (1 - I_t)\, z_{t-1} + \varepsilon_t \quad [\, + \ \textstyle\sum_{j=1}^{k} \gamma_t \cdot \Delta z_{t-j}]$$

using the residuals $\hat{z}_t$ from the first step instead of the true errors z_t and test the null hypothesis H_{CI}^0: $\varrho_+ = 0 \ \wedge\ \varrho_- = 0$ (no co-integration) against the alternative H_{CI}^A: $\varrho_+ < 0 \ \wedge\ \varrho_- < 0$ (co-integration). Enders and Siklos suggest conducting an *F-test with non-standard critical values* because under H_0 the standard (asymptotic-normal) theory does not apply. The critical values for the Enders-Siklos CI-test are much larger than those from standard theory, e.g. at the level of 5%, the 95%-quantile of the $F_{2,T-2}$distribution is approximately 3, while the proper critical value for the Enders-Siklos CI-test is roughly 6.

To test for an APT, Enders and Siklos suggest conducting a second F-Test: H_{APT}: $\varrho_+ = \varrho_-$ (null is no APT, i.e. symmetry) against the alternative H_{APT}^A: $\varrho_+ \neq \varrho_-$ (the alternative is APT, i.e. asymmetry) using standard critical values coming from the $F_{1,T-1}$ distribution. Note that this test is invalid without co-integration. In a sense, the CI-test has the role of a diagnostic test to justify applying the APT-test.

Remark: Note that the test statistic for the asymmetry test can be written as:

$$F_{APT} = \frac{1}{T-1} \frac{SSE_{\,sym} - SSE}{SSE},$$

where SSE is the sum of squared residuals of the error regression (allowing $\varrho_+ \neq \varrho_-$) and $SSE_{\,sym}$ is the analogous quantity assuming symmetry (i.e. under the restriction $\varrho_+ = \varrho_-$).

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B. Extension of the Enders-Siklos procedure: estimate threshold τ

So far it was assumed that threshold τ is known in advance. As this will usually not be the case, Enders and Siklos suggest estimating τ by running the residual regression for many potential τ's except for the e.g. 15% smallest and largest of these values, and taking the one which best fits the data (minimizes the SSE). The estimation of τ leads to new critical values for the CI-test (5% critical value now $F_{CI}^{*}(5\%) \approx 7$).

Ender and Siklos are vague about the statistical inference with the asymmetry test under the extension. However, in this work the test for asymmetry was conducted the same way as with known τ (i.e. retaining the standard errors and critical values from the F-distribution). This proceedure is not as innocent as it might appear, because the estimation of τ leads to a new distribution of the estimated reversion rates. The major difficulty occurs precisely under the null of asymmetry test, in that threshold τ, as a parameter to be estimated, is not identified in this case. Under the null of the asymmetry test, the SETAR/MTAR error model reduces to an AR-model, upper (ϱ_+) and lower (ϱ_-) regime are indistinguishable. They merge into a single regime and threshold τ becomes irrelevant: any τ results in the same model for the CI-errors. As long as τ is specified, this is unproblematic but if τ is estimated, the asymmetry test turns out to be massively oversized if standard errors and critical values are retained. Naturally, one might resort to bootstrapping the estimated ($\varrho_{-,}\varrho_+$), but with the 'brute force search' of the 'optimal' τ this is very costly in terms of computational time and is rarely done in practice.

Remark: Note that estimating τ by minimizing SSE is equivalent to maximizing F_{APT} (over the considered range of potential thresholds), which is apparent from the above formula for F_{APT}. Therefore the estimation of τ will always result in 'more significance for asymmetry', if the test for asymmetry is conducted as in the case of the given τ.

C. Demonstration: weekly crude oil and gasoline prices (Germany, Jan 2004 – June 2012)

The paper applied the Enders-Siklos procedure to German crude oil and gasoline prices. The results were obtained using the R-package APT, which provides a ready-to-use implementation of the Enders-Siklos procedure (conducting an unmodified F-test in case τ is estimated).

Asymmetry in price transmissions is found to be significant at the 5% level only if the MTAR model with estimated τ is used. In view of the simulation results in the next section, this significance should be doubted.

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Bootstrapping the error regression (not performed) is supposed to lead to non-significance of APT in these data also for the MTAR(τ^*) model.

	SETAR	SETAR(τ^*)	MTAR	MTAR(τ^*)	crit. val.(5%)
T (# obs.)	447	447	447	447	
k (# lags)	1	1	1	1	
τ	0	–0.034	0	–0.015	
ϱ_+	–0.115	–0.103	–0.087	–0.097	
(t.val. ϱ_+)	(–3.33)	(–3.44)	(–2.54)	(–3.61)	≈ 2
ϱ_-	–0.121	–0.146	–0.150	–0.217	
(t.val. ϱ_-)	(–3.54)	(–3.55)	(–4.35)	(–3.87)	≈ 2
AIC	1619.87	1619.16	1618.12	1616.02	
BIC	1636.26	1635.55	1634.51	1632.41	
F.val. H_0: “no CI”	11.38	11.75	12.23	13.40	≈ 6
F.val. H_0: “no APT”	0.02	0.72	1.76	3.86	$\approx 1.96^2 = 3.84$
p.val. H_0: “no APT”	0.90	0.40	0.19	0.05	

2. Simulation results

The aim was to check the validity of the test for asymmetry in the Enders-Siklos procedure, assuming that the true situation is ‘symmetric co-integration’: the author simulated samples under $H_{CI}^{A} \wedge H_{APT}^{0}$, which means that errors z_t followed a stationary AR-process. The results presented refer to the case $\Delta z_t = \varrho \cdot z_{t-1} + \epsilon_t$, where $\varrho = -0.1$ and $\epsilon_t \sim i.i.d.\ N(0,1)$. The following additional assumptions and simplifications were imposed (the results do not change substantially if they are dropped, see the paper):

- directly model the CI-errors instead of the residuals,
- assume no (further) autocorrelation in CI-errors (assume $k = 0$, which is known),
- for simplicity, $F_{CI}^{*} = 6$ is used as the 5% critical value of the CI-test throughout.

The correct values are slightly larger, which makes the results even more extreme.

All the tests were performed at the 5% level; for the symmetry test in the case of threshold estimation no provision was made to account for the change in distribution of the reversion rates.

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The following table shows the simulation results obtained after 20 000 replications for

$P_{CI} = P(F_{CI} > F^*_{CI})$ = Probability of (correctly) identifying co-integration = Power of the CI-test.

$P_{APT} = P(F_{APT} > F^*_{APT})$ = Probability of (falsely) detecting asymmetry, ignoring result of CI-test = Size of the unconditional test for asymmetry (should be ≈ 5%).

$P_{APT|CI} = P(F_{APT} > F^*_{APT} \mid F_{CI} > F^*_{CI}) =$ Probability of (falsely) detecting asymmetry conditional on 5% significance for CI = Size of the conditional test for asymmetry.

	SETAR	SETAR(τ^*)	MTAR	MTAR(τ^*)
$\varrho = -0.1, T = 100$				
$P_{CI} = \frac{n_{CI}}{n_{all}}$	$0.09 = \frac{1\,880}{20\,000}$	$0.18 = \frac{3\,619}{20\,000}$	$0.09 = \frac{1\,765}{20\,000}$	$0.24 = \frac{4\,852}{20\,000}$
$P_{APT} = \frac{n_{APT}}{n_{all}}$	$0.06 = \frac{1\,231}{20\,000}$	$0.21 = \frac{4\,163}{20\,000}$	$0.05 = \frac{950}{20\,000}$	$0.32 = \frac{6\,302}{20\,000}$
$P_{APT\|CI} = \frac{n_{APT \wedge CI}}{n_{CI}}$	$\mathbf{0.28} = \frac{517}{1\,880}$	$\mathbf{0.53} = \frac{1\,934}{3\,619}$	$\mathbf{0.24} = \frac{664}{1\,765}$	$\mathbf{0.65} = \frac{3\,154}{4\,852}$
$\varrho = -0.1, T = 200$				
$P_{CI} = \frac{n_{CI}}{n_{all}}$	$0.49 = \frac{9\,780}{20\,000}$	$0.64 = \frac{12\,850}{20\,000}$	$0.48 = \frac{9\,516}{20\,000}$	$0.73 = \frac{14\,621}{20\,000}$
$P_{APT} = \frac{n_{APT}}{n_{all}}$	$0.06 = \frac{1\,223}{20\,000}$	$0.20 = \frac{3\,924}{20\,000}$	$0.05 = \frac{924}{20\,000}$	$0.34 = \frac{6\,874}{20\,000}$
$P_{APT\|CI} = \frac{n_{APT \wedge CI}}{n_{CI}}$	$\mathbf{0.11} = \frac{1\,120}{9\,780}$	$\mathbf{0.28} = \frac{3\,608}{12\,850}$	$\mathbf{0.09} = \frac{849}{9\,516}$	$\mathbf{0.38} = \frac{6\,342}{14\,621}$

The data come from a symmetric co-integration. Both the CI- and the APT-tests were conducted at 5% level.

P_{CI} = Power of the CI-Test

P_{APT} = Size of test for asymmetry, ignoring the result of the CI-test;

$P_{APT|CI}$ = Effective size of the test for asymmetry under 5% significance for CI.

The results concerning P_{CI} (first rows) show the low power of the CI-test, which improves as the sample size increases. The results concerning P_{APT} (second rows) indicate that the symmetry test complies with its significance level (5%) when the threshold is exogenously given. However, if τ is estimated, the symmetry test is strongly oversized (i.e. invalid: it refuses its null of symmetry far too frequently for a test at the 5% level). Note that this oversizing does not appear to decline with sample size, with the MTAR-model it even appears to increase.

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3. Conclusion

In the situation of a symmetric cointegration, the usual implementation of the Enders-Siklos procedure is heavily biased towards finding asymmetry. This bias can be attributed to two sources. The first one is the selection effect: in this work, the test for asymmetry was conducted only if the cointegration was sufficiently significant in the data, which has the side effect that the estimated reversion rates from above and below tend to differ significantly far more frequently than by chance. Similar selection effects occur with any test which is conducted only if a primary diagnostic test is successful. However, with the Enders-Siklos procedure the selection bias was particulary strong because of the low power of the CI-test to detect the cointegration (as reflected by its extraordinarily large critical values).

The second contribution to the bias comes from an identification problem which manifests itself only if the threshold for the SETAR or MTAR-model is considered to be a parameter to be estimated (the usual practice applied). With a perfectly symmetric cointegration, the threshold is completely undetermined; estimating it (without accounting for this in the standard errors of the reversion rates) favours finding 'significance for asymmetry'. If the usual practice is applied with an almost symmetric cointegration, a much too high (indicated) significance for asymmetry results. As opposed to the selection effect, the bias arising from the identification problem persists for large sample sizes.

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