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NUMERICAL MODELLING FOR UNDERGROUND MINING RELATED GEOTECHNICAL ISSUES

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Abstract: Issues presented in this work relate to geotechnical problems that are specific to the mining areas. The paper discusses the methodology of mathematical and numerical modeling of these problems. Examples contained in the paper include: predicting the influence of mining exploitation on a detached building and evaluating the effectiveness of the building protection with the trench. Possible applications of numerical modeling as a tool to aid the continuous monitoring of the building state during the exploitation have also been discussed.

1. INTRODUCTION

A certain category of geotechnical issues can be observed in areas affected by underground mining, which are related to the disturbances in the original structure of the rock mass. As a result of mining, a building subsoil is subject to deformation processes variable in time and space, imposing an additional load on the building. Depending on building location in relation to a specific field, subsoil deformations may be transient or permanent. Permanent deformation zones can be observed in areas over the exploitation edges. In the central areas, the surface deformations, apart from the permanent subsidence, are in theory, only transient.

Underground mining related geotechnical issues result from the deformation processes in the subsurface of the rock mass layers, defined as forming of a mining subsidence trough. The process is described considering the geometry by Budryk–Knothe's theory, commonly used in Poland [3], [4]. The basic equations of the theory determine surface subsidence as a function of the geometrical coordinates and time. Assuming the definitions of tilt and curvature as derivatives of the subsidence function and assuming the proportionality of horizontal displacement to the tilt, distribution and values of the deformation indices may be determined. The three indices: tilt, radius of curvature and horizontal strain are the basic measure of the mining induced effects. Predicting values of the indices forms the basis for land classification into a mining area category. The following categories indicate the hazard level for the surfaces affected by mining induced deformations – the higher the category, the higher the deformation hazard [5].

2. PREVENTION OF MINING DAMAGE IN BUILDINGS

A prevention of mining damage in buildings is based on the predicted influence of the mining induced land deformation on the building structure, although not only the bearing capacity, but also the serviceability is considered. The existing buildings are not usually fitted with means of protection allowing safe transfer of mining deformations from the subsoil. The main issue at the building design stage is that it is not possible to anticipate the type of mining induced impact on the building throughout its service life. Mining extraction plans are developed and approved for several years, which only covers a part of the service life of a building. An assessment of the impact of mining on buildings and utilities is carried out at the mining exploitation design stage. The tools to assess the impact on typical masonry structures include local building resistance assessment methods enabling estimated resistance evaluation [5], [8]. The buildings classified into hazardous group may require structural or geotechnical protection and/or constant technical supervision in the course of showing the main impact of mining.

The typical prevention means are more often assisted by the advanced analytical tools. Numerical modelling based on complex mathematical models enables numerical simulation of the impact of planned or completed mining extraction on the building and its subsoil. As a result of the analysis, we obtain an answer to the question on the impact of mining on the buildings, or the question if the existing damage might have been induced by mining. Numerical modelling is also used in the design of building protection means against the impact of mining. It may also be used as a tool to aid building condition monitoring in the course of extraction.

3. NUMERICAL MODELLING AS A TOOL FOR PREDICTING BUILDING CONDITION

Numerical modelling allows a detailed analysis of the impact of mining in a specific case. A formulation of a mathematical model of a building and its subsoil, allowing for the number of factors affecting the condition of the subsoil and the building structure is required to complete the analysis [2]. The factors can be classified into three basic data sets, including:

- mining conditions and history,
- geological and geotechnical conditions,
- building structural conditions.

Collecting data of a required quantity and quality requires not only a thorough analysis of the technical, geological and mining documentation, but also quantity surveys, site visits, laboratory and *in situ* testing. Although it involves labour and costs, it is an indispensable basis for a correct mathematical model.

A key model element is to develop a method of mining influence simulation. The model including a rock mass area with mined bed is difficult to develop due to the size of the task. The issue can be solved using a *submodelling* method. The alternative method is to consider only the section of the rock mass being the direct base for the building and subject to exploitation. The dimensions of the section are selected at the initial calculation stage to neglect the impact of the building on the stress state outside the section and to eliminate the effect of disturbances from boundary conditions in the analysed area. The impact of planned mining extraction can be simulated in this model using, for example, Budryk–Knothe's theory equations [3], [4]. The mining induced changes in the construction and the subsoil state can be predicted using this method.

3.1. EXAMPLE 1. PREDICTION OF STRESS STATE IN THE DETACHED BUILDING

The detached building located over the middle section of longwall at a depth of 455 m was analyzed. Exploitation depth is approx. 2.1 m, and the average exploitation speed is 3 m/day. For the purposes of the object model it was necessary to carry out the following: analysis of mining history and conditions, construction and geotechnical conditions of the building and subsoil and laboratory tests performed on samples of rock, soil and building materials. A physical and mathematical model of the building–subsoil system was formulated on this basis. In the four-dimensional spacetime with geometric Cartesian coordinate system 1, 2, 3 this model is written as a set of equations:

1) equations of motion

$$S^{(1)}{}_{ij,j} + \rho_0 b_i = \rho_{_0} \ddot{u} \,,$$

2) kinematics equations

$$E_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) \,,$$

3) constitutive equations

$$\sqrt{(d'|_0 - p_t|_0 \operatorname{tg}\beta_s + q^2)} - p \cdot \operatorname{tg}\beta_s - d' = 0 \quad \text{for the bottom layer of the subsoil,}$$

$$R_{mc} q - p \cdot \operatorname{tg}\phi - c = 0 \quad \text{for the upper layer of the subsoil,}$$

where

$$R_{mc}(\theta,\phi) = \frac{1}{\sqrt{3}\cos\phi}\sin\left(\theta_s + \frac{\pi}{3}\right) + \frac{1}{3}\cos\left(\theta_s + \frac{\pi}{3}\right)\operatorname{tg}\phi \,,$$

$$\cos(3\theta_s) = \left(\frac{r_s}{q}\right)^3,$$

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \qquad \text{for the building,}$$

where

$S_{ij}^{(1)}$ – component of the nominal stress tensor,
S_{ij} – component of the deviator,
σ_{ij} – component of the Cauchy stress tensor,
E_{ij} – component of the Green–Saint-Venant strain tensor,
ε_{ij} – component of the Cauchy strain tensor,
ε_{kk} – volumetric strain,
u_i – component of the displacement vector,
b_i – component of the body force vector,
p – equivalent pressure stress; $p = -\frac{1}{3}\sigma_{ii}$,
$p_t|_0$ – initial hydrostatic tension strength,
q – the Mises equivalent stress; $q = \sqrt{\frac{3}{2}S_{ij}S_{ij}}$,
r_s – the third invariant of deviatoric stress; $r_s = \left(\frac{9}{2}S_{ij}S_{jk}S_{ki}\right)^{\frac{1}{3}}$,
ρ_0 – the initial value of density,
d' – the hardening parameter,
G – shear modulus; $G = \frac{E}{2(1+\nu)}$,
λ – the Lamé constant; $\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}$,
E – Young's modulus,
ν – Poisson's ratio,
β_s – friction angle measured at high confining pressure,
ϕ – angle of friction,
c – cohesion,
θ_s – the Lode angle;

4) kinetic boundary conditions at the unloaded boundary

$$S_{ij}^{(1)}\, n_j = 0,$$

5) kinematic boundary conditions at the boundary where the impact of exploitation was applied

$$u_i = f(X_i, t)$$

where

X_i – geometrical coordinate (i = 1, 2, 3),

t – time;

6) geostatic initial conditions in the subsoil

$$S_{33}^{(1)} = \gamma h,$$
$$S_{11}^{(1)} = S_{22}^{(1)} = \mu S_{33}^{(1)},$$

where

1, 2, 3 – the Cartesian coordinate system in which 3 is the vertical direction,

γ – specific weight of the subsoil,

h – thickness of the subsoil layer,

μ – coefficients of lateral stress;

7) contact conditions at the interface between the foundation and walls of the cellar to the subsoil

$$\left.\begin{aligned} c_n &\geq 0 \\ N_n &\leq 0 \\ c_n N_n &= 0 \end{aligned}\right\}$$

where

c_n – is the distance between the contact surfaces,

$\boldsymbol{N}$ – the contact stress tensor,

$\boldsymbol{N_n}$ – the normal contact stress tensor,

$\boldsymbol{n}$ – the unit normal vector,

then,

– normal contact

$$\boldsymbol{N}_n = (\mathbf{S}^{(1)}\boldsymbol{n})\boldsymbol{n},$$

– tangential contact

$$\left|\boldsymbol{N}_s\right| < f\boldsymbol{N}_n, \qquad \dot{\boldsymbol{u}}_N = 0,$$
$$\left|\boldsymbol{N}_s\right| = f\boldsymbol{N}_n, \qquad \dot{\boldsymbol{u}}_N = -\chi\boldsymbol{N}_s,$$

$$\boldsymbol{N}_s = \boldsymbol{N} - \boldsymbol{N}_n,$$

where

$\boldsymbol{N_n}$ – normal components of the contact stress,

$\boldsymbol{N_s}$ and $\boldsymbol{N_t}$ – tangential components of the contact stress,

$\boldsymbol{u}_N$ – contact displacement vector,
f – friction coefficient.

The material properties assumed for this analysis were determined from the results of laboratory tests on samples of construction and geological materials. The data are listed in Table 1.

Table 1

The material properties assumed for analysis

Material	Model	Parameters of the model
the bottom layer of the subsoil–rock material	extended Drucker–Prager model in hiperbolic form	E = 16.8 GPa ν = 0.12 β_s = 58°, d' = 21 MPa, l_0 = 18.9 MPa ρ = 2 500 kg/m^3
the upper layer of the subsoil–soil material	Mohr–Coulomb model	E = 0.12 Gpa ν = 0.3 ϕ = 36°, c = 0 ρ = 2 000 kg/m^3
the masonary	linear elasticity model	E = 6.0 GPa ν = 0.3 ρ = 1 400 kg/m^3
floors	linear elasticity model	E = 200.0 GPa ν = 0.3 ρ = 2 800 kg/m^3

Abaqus FEM software has been used for numerical simulations. The prediction of the longwall 133 exploitation influence on the building is based on Budryk–Knothe's theory used as a mathematical model of the subsoil deformation. Therefore, the function $f(X_i, t)$ in the kinematic boundary conditions takes the form which results from equations of this theory. These conditions were imposed by subroutine included in Abaqus/Standard solver. In this way a numerical simulation carried out. A detailed description of the building with a wider discussion of the modelling process is included in monograph [2][1]. The calculation results allow determination of values and directions of stress, strain and displacement for individual structure components at each exploitation stage.

Figure 1 shows changes in the principal stress state resulting from the mining induced subsoil deformations. The adverse effect of negative deformation associated with the process of soil compaction when the building is located in the concave part of the trough can be observed. Significant minimum values of the principal stress on the basement are associated with an increase in maximum principal stress values on the higher storeys. In the case analyzed the values of these influences were not dangerous

[1] The tests have been carried out as part of MNiSW N524 1749 33 research project.

for the object. Geometric shape of the building in the form of a compact body, the subsoil, which suppressed the mining deformation and good technical condition are factors that significantly reduced the risk of damage. This prediction is confirmed by the observation of the building during the exploitation of the longwall 133. In the construction no mining damage is observed [2].

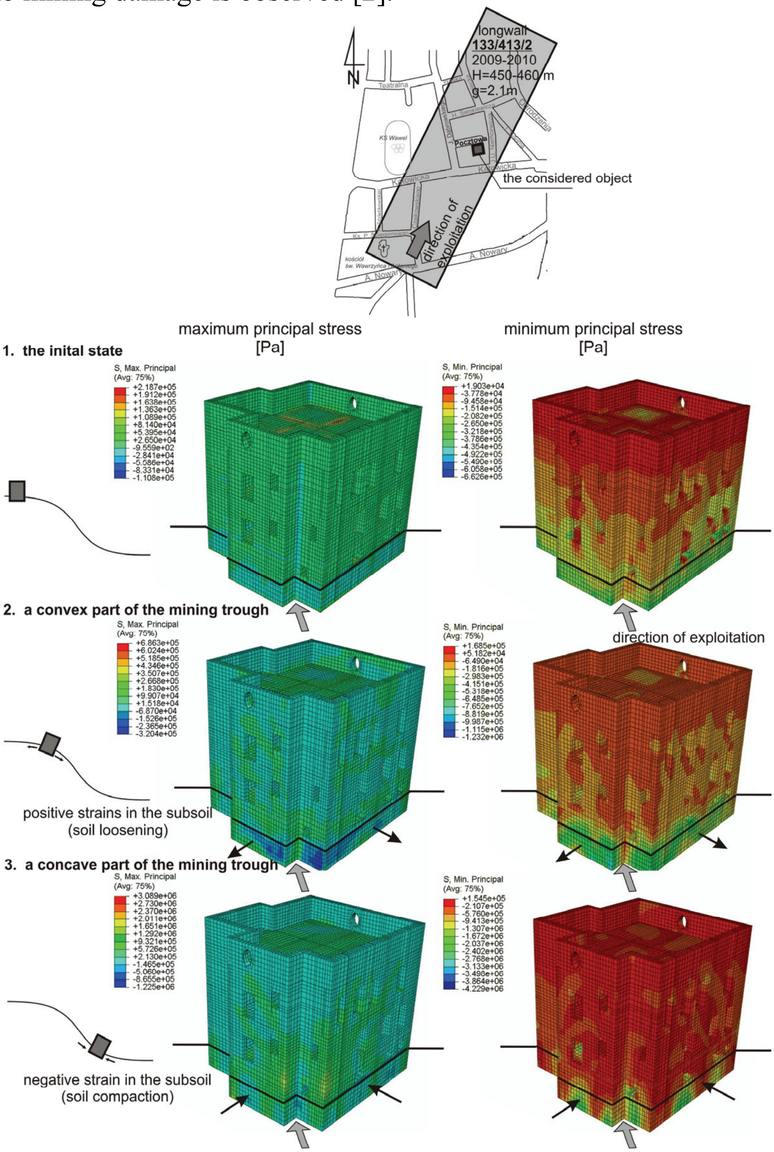

Fig. 1

4. NUMERICAL MODELLING IN THE DESIGN OF BUILDING PROTECTION

Numerical modelling can also be used as an aid to the design of means of building protection. It allows, among other things, an analysis of the efficiency of the designed mining and construction preventive means.

Both the experience and the calculation results show that among the indices of mining surface deformations shown at the beginning of the article, a negative subsoil strain has the most adverse impact on buildings [8], [2]. One of the methods to protect the construction is the trench dug around the building as a geotechnical method to reduce the adverse impact of mining [6], [1].

4.1. EXAMPLE 2. ASSESSMENT OF THE EFFECTIVENESS OF THE GEOTECHNICAL MEANS OF BUILDING PROTECTION

Analysis of the impact of trenches on the state of strain in building subsoil subjected to the influence of underground mining has been carried out on the theoretical example. A scheme of the problem is shown in Fig. 2. The task was formulated as a contact problem of linear elasticity theory in the form of the following equations system:

1) equations of motion

$$\sigma_{ij,j} + \rho_0 b_i = \rho_0 \ddot{u},$$

2) kinematics equations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

3) constitutive equations

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij},$$

4) kinetic boundary conditions at the unloaded boundary

$$S_{ij}^{(1)} n_j = 0,$$

5) kinematic boundary conditions at the boundary where the influence of exploitation was applied

$$u_i = f(X_i, t),$$

6) geostatic initial conditions in the subsoil

$$\sigma_{33} = \gamma h,$$
$$\sigma_{11} = \sigma_{22} = \mu\sigma_{33},$$

7) contact conditions

$$\left.\begin{aligned} c_n &\geq 0 \\ N_n &\leq 0 \\ c_n N_n &= 0 \end{aligned}\right\}$$

– in the normal direction *n*

$$\boldsymbol{N}_n = (\boldsymbol{\sigma n})\boldsymbol{n}\,,$$

– in the tangential direction *s*

$$\begin{aligned} |\boldsymbol{N}_s| &< f\boldsymbol{N}_n, & \dot{\boldsymbol{u}}_N &= 0, \\ |\boldsymbol{N}_s| &= f\boldsymbol{N}_n, & \dot{\boldsymbol{u}}_N &= -\chi\boldsymbol{N}_s, \end{aligned}$$

$$\boldsymbol{N}_s = \boldsymbol{N} - \boldsymbol{N}_n\,,$$

where $\dot{\boldsymbol{u}}_N$ is a slip rate

$$\dot{\boldsymbol{u}}_s = \frac{\partial \boldsymbol{u}_s}{\partial t}\,.$$

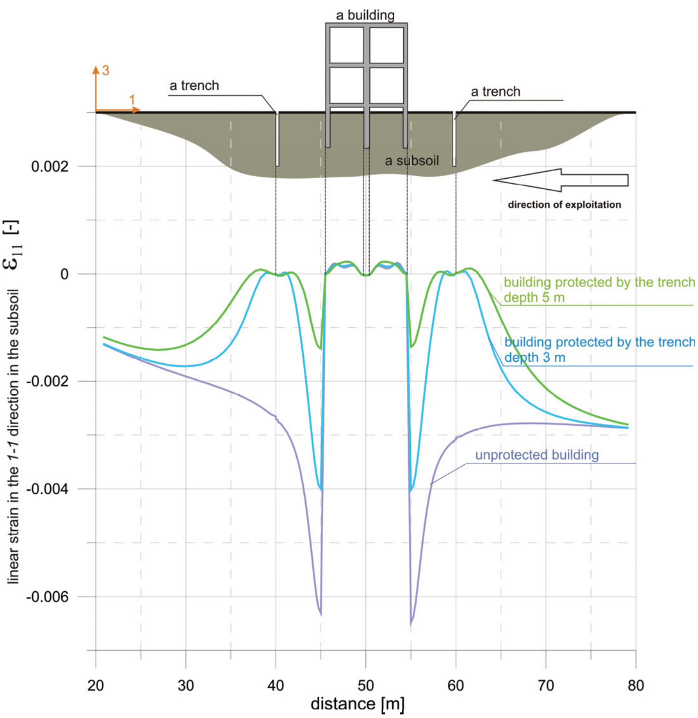

Fig. 2

Theoretical material models that represent real media were used in the example. Their behavior is described by the Hook law. The material properties taken for the calculation are listed in Table 2. Abaqus FEM software has been used for numerical simulations

Table 2

The theoretical material properties assumed for analysis in example 2

Theoretical material	Density ρ [kg/m^3]	Young's modulus E [MPa]	Poisson's ratio ν [–]
the bottom layer of the subsoil (soil material)	2 000	0.12E+09	0.3
the upper layer of the subsoil (rock material)	2 500	16.8E+09	0.12
masonary	1 600	10E+09	0.25
reinforced concrete floor	2 500	20E+09	0.28
material that fills the trench	2 000	2E+03	0.33

The calculations allow the assessment of the reduction in adverse impact level for trenches of various depths[2]. These results are shown in Fig. 2. The calculations provided that the presence of the trench dug around the building with a depth of 3 m would reduce the compressive strain mine in the subsoil by about one third. The trench with a depth of 5 m will reduce the adverse mining impacts by over 70% compared to the absence of such protection.

5. NUMERICAL MODELLING IN CONSTRUCTION STATE MONITORIN AND IN FAILURE CAUSE ANALYSIS

One of the preventive measures is a continuous monitoring of the construction state in the period of the most adverse impact of the mining exploitation. Apart from standard technical supervision methods, automatic monitoring methods are increasingly used [7]. Automatic monitoring methods are most often based on the vibrating wire extensometer or systems based on advanced geodetic techniques measuring changes in point coordinates. The method provides information on changes at selected structural points (sensor installation points).

Use of mathematical models and measuring system gives an overall image of the structure performance. A correlation of the measurement results and calculation results is obtained using virtual “sensors” within the structural model, where measuring system sensors are installed [2]. The boundary conditions simulating mining

[2] The tests have been carried out as part of MNiSW N N524 466636 research project.

induced subsoil deformations shall be determined based on current geodetic measurements of the land surface deformation. A prerequisite for reliable results is a correct calibration of the model, assuming that the conformity of the calculation results and the measurement results for specific points implies the correct solution for the entire area.

Numerical simulation is also an effective tool in the analysis to determine the cause of existing construction damage. As for the monitoring, mining induced deformation modelling shall be based on the measurement of actual mining induced land surface deformation. The calculation results may be used to determine the changes in stress and deformation of the construction induced by mining, which is also the basis for the assessment.

6. SUMMARY

The problems presented in the article concern geotechnical issues typical of the areas subject to underground mining. Within the areas, the constructions are burdened with an additional group of land deformation related interactions. The study presents brief specification of standard building prevention methods and possible use of analytical methods. It discusses a numerical modelling method of the analysis of the impact of mining on buildings and use of numerical modelling in the assessment of planned or completed mining on the building structure and its subsoil. It also presents the possible use of the method in the analysis of the efficiency of planned protection means and building state monitoring in the course of showing the main impact of mining. The issues are illustrated with calculation examples.

The study presents numerical modelling methods based on complex mathematical models, as a tool for scientific and engineering analyses of a certain category of geotechnical issues. Moreover, it also discusses suggested use of the developed method for protection of buildings in the areas affected by the underground mining.

REFERENCES

[1] CAUDRON M., HOR B., HEIB M., EMERIAULT F., *Effectiveness of a mitigation technique for buildings subjected to ground subsidence*, International Journal of Geotechnical Engineering, Issue 2/April 2012 – Advances in Geotechnical Engineering, 179–184(6).

[2] FLORKOWSKA L., *Zastosowanie numerycznej mechaniki nieliniowej w zagadnieniach ochrony budynków na terenach górniczych*, Archives of Mining Sciences, Monografia nr 11, Kraków 2010.

[3] KNOTHE S., *Równanie profilu ostatecznie wykształconej niecki osiadania*, Archiwum Górnictwa i Hutnictwa, Tom 1, Zeszyt 1, 1953, 111–127

[4] KNOTHE S., *Wpływ czasu na kształtowanie się niecki osiadania*, Archiwum Górnictwa i Hutnictwa, Tom 1, Zeszyt 1, 1953, 128–139.

[5] KWIATEK J. (ed.), *Ochrona obiektów budowlanych na terenach górniczych*, Wyd. GIG.

[6] MISA R., TAJDUŚ K., SROKA A., *Numeryczna analiza metod geotechnicznych minimalizujących wpływ przemieszczeń powierzchni terenu wywołanych eksploatacją podziemną*, Prace Instytutu Mechaniki Górotworu PAN, Tom 13, nr 1–4, 2011, 81–93.
[7] WITAKOWSKI P., *Bezprzewodowy monitoring obiektów budowlanych*, Materiały IX Sympozjum "System kompleksowego zarządzania jakością w budownictwie", ITB, Warszawa 2010.
[8] WODYŃSKI A., *Ocena odporności budynków murowanych na wpływy górnicze w świetle badań i doświadczeń*, [w:] *Ochrona obiektów na terenach górniczych*, praca zbiorowa, Katowice 1997.