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EFFICIENCY MEASUREMENT USING NONPARAMETRIC PRODUCTION ANALYSIS IN THE PRESENCE OF UNDESIRABLE OUTPUTS. APPLICATION TO POWER PLANTS

In order to deal with undesirable products in models of performance analysis, we need to replace the assumption of free disposability by weak disposability and this assumption has been used to model undesirable products as outputs. The traditional axiom of weak disposability of Shephard (1970) is given in a multiplier form and, in this sense, the level of bad outputs is equal to zero if and only if the level of the desirable outputs is equal to zero. An alternative definition of the weak disposability of outputs in additive form has been proposed. An axiomatic foundation has been introduced to construct a new production technology space in the presence of undesirable outputs. The model is illustrated using real data from 92 coal fired power plants.

Keywords: *data envelopment analysis, performance analysis, undesirable outputs, weak disposability, efficiency, input/output.*

1. Introduction

Performance measurement is an important task for operational units, in order to find their weaknesses and strengths and thus proscribe subsequent improvements. To this end, it is important to know the structure of the production technology. Over the past four decades, many authors have developed techniques to assess production technology in a way that is consistent with the underlying economic theory of optimal behaviour.

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Performance measurement, using tools such as data envelopment analysis (DEA), makes it possible to consider feasible production plans and trade-offs between inputs and outputs based on the empirical characterization of a production technology set. Since the pioneering work of Charnes et al. [1], DEA has been established a robust and valuable methodology for frontier estimation. It is a widely used method for analysing technical efficiency and based on this technique, a number of additional applications have been suggested for supporting the planning of efficiency improvement.

The applicability of DEA depends on the underlying production technology set and the consistency of this model with the properties of the technology is important. In production theory, technologies are constructed axiomatically and based on the construction of the technology set, an appropriate model is constructed and used to evaluate the relative performance of decision making units (DMUs).

The original DEA model of Charnes et al. [1] (the CCR model) assumed that all outputs are desirable and decision makers would like to increase production of these good outputs and to decrease the level of inputs. However, the production process often results in undesirable outputs and we normally want to reduce the level of these bad outputs. So, there is a need to provide an alternative method of evaluating the relative performances of DMUs in the presence of undesirable outputs.

In the last two decades, modelling undesirable outputs in a production processes has attracted considerable attention. There is a group of papers proposing methods to handle undesirable outputs in a production process (e.g., [2, 14, 15]. Hailu and Veeman [3] treated undesirable outputs as inputs and used a classical DEA model to evaluate the relative efficiency of DMUs in the presence of desirable and undesirable outputs. As Färe and Grosskopf [4] stated, considering undesirable outputs as inputs is inconsistent with the laws of physics and the standard axioms of production theory. In order to deal with undesirable outputs in models of performance analysis, we need to replace the assumption of the free disposability of outputs by weak disposability. They used the weak disposability assumption of Shephard [16] to model undesirable products as outputs. Färe and Grosskopf [4] were the first to propose a DEA based approach by imposing an assumption that bad outputs are weakly disposable. They applied a single abatement factor to all the firms observed in a sample. However, the use of this single abatement factor affects the production set. Later, Kuosmanen [12] argued that the correct implementation of the axiom of weak disposability requires the use of different abatement factors for each firm. He presented a simple formulation of weak disposability that allows non-uniform abatement factors and preserves the linear structure of the model. Despite Färe and Grosskopf [8] claiming that a single abatement factor suffices for modelling weak disposability in a nonparametric production model, Kuosmanen and Podinovski [11] demonstrated that a single abatement factor does not suffice to capture all feasible production plans and they proved that Kuosmanen's [10] technology set is the correct and complete minimum technology set. Podinovski and Kuosmanen [13] studied the problem of weak disposability in DEA under a relaxed assumption regarding convexity.

We believe that if we want to model undesirable outputs as outputs, instead of inputs, the rational and correct treatment is to use the assumption of weak disposability. The existing definition of weak disposability is given by Shephard [16] and this assumption is given in a multiplier form and, in this sense, the level of undesirable outputs would be equal to zero if and only if the level of desirable outputs is equal to zero. However from experience we know that in real applications we confront cases in which, up to a certain point, a certain amount of desirable outputs are produced without any undesirable outputs. On the other hand, above this threshold, undesirable outputs are produced along with desirable outputs. This means that by consuming a certain amount of inputs, we can produce $(y, 0)$ as a mix of desirable and undesirable outputs, respectively, where $y > 0$.

As an example of such a process, consider performance evaluation in the branches of a bank. In the banking sector in Iran, we can define two outputs: loans as desirable outputs and overdue debts as undesirable outputs. Clearly, by using strict criteria for granting loans, we may give loans to customers without any overdue debts resulting as an effect. However, when the criteria for granting loans are relaxed, overdue debts begin to appear.

As another simple example, consider a brick-kiln that uses coal to generate the heat that is used to produce bricks. Clearly, by burning the coal, pollutants are also produced and the temperature of the brick-kiln must reach a certain degree to produce any bricks. Hence, in this case, some level of undesirable output must be produced before any desirable output is achieved.

These two simple examples show that zero undesirable (desirable) outputs do not necessarily require zero desirable (undesirable) outputs. In this paper, we propose an alternative definition of weak disposability in an additive form. This new definition applies a fixed reduction factor to each input/output measure. This reduction factor decreases the level of undesirable factors by decreasing the activity level according to an additive form. Based on this definition of the weak disposability of outputs, undesirable outputs are modelled as outputs instead of inputs. An axiomatic foundation is introduced to construct a new production technology set and the linear structure of the new technology set is preserved. The new production technology set is fundamentally different from those proposed by previous approaches. So we should not expect to have the same efficient frontiers as those defined by other approaches and, in this case, a firm may be inefficient according to our approach, while it is efficient according to another approach.

The structure of the paper is organized as follows: In the next section, a new definition of weak disposability is given. A real application based on 92 coal-fired power plants is given in section three. The conclusions appear in section four.

2. Weak disposability

Suppose we have K DMUs and each $DMU_k: k = 1, 2, ..., K$ consumes N inputs to produce M desirable outputs and J undesirable outputs. The input vector of DMU_k is represented by $x^k = \left(x_1^k, x_2^k, ..., x_N^k \right)$. The vectors of desirable and undesirable outputs are represented by $v^k = \left(v_1^k, v_2^k, ..., v_M^k \right)$ and $w^k = \left(w_1^k, w_2^k, ..., w_J^k \right)$, respectively. The production technology T is characterized by the production set

$$T = \left\{ (x, v, w) \,\middle|\, x \text{ can produce } (v, w) \right\}$$

An equivalent representation of this set in terms of the output set is

$$P(x) = \left\{ (v, w) \,\middle|\, (x, v, w) \in T \right\}$$

In order to model undesirable outputs in the performance analysis, we should replace the assumption of free disposability of outputs by weak disposability. A formal definition of weak disposability is given by Shephard [16]:

Definition 1. Outputs are weakly disposable if $(v, w) \in P(x)$ and $0 \leq \theta \leq 1$ implies

$$(\theta v, \theta w) \in P(x)$$

An interpretation of this definition is that scaling down the level of bad outputs results in a reduction in the level of good outputs by the same proportion. The abatement factor θ indicates how the level of undesirable outputs, w, is scaled down when the activity level falls. Suppose that, for example, we are producing electricity by burning coal. Clearly, in this process the bad output is the carbon dioxide that is produced along with electricity. Shepherd's definition of weak disposability implies that a θ% reduction in carbon dioxide emissions is feasible if there is a θ% reduction in electricity production (under the assumption of a constant input vector).

This definition of weak disposability is given in a multiplier form. In this sense, the level of undesirable outputs would be equal to zero if and only if $\theta = 0$ and in this case, the level of desirable outputs is also equal to zero. However, production processes can be found in which this production rule does not hold true. We may encounter cases in which undesirable products are produced only after producing a certain amount of desirable outputs. In other words, it may be possible to produce the output vector $(y, 0)$, where $y > 0$ is the level of desirable output and the zero component corresponds to the level of undesirable output. So, we need a new definition of weak disposability to be applicable in such a case. In what follows, we provide an alternative definition of weak disposability in additive form.

Definition 2. Outputs are weakly disposable if $(v, w) \in P(x)$ and $(\alpha_M, \alpha_J) \geq 0$ imply that $0 \leq (v - \alpha_M, \ w - \alpha_J) \in P(x)$ where $\alpha_M = (\alpha, \alpha, ..., \alpha)$ and $\alpha_J = (\alpha, \alpha, ..., \alpha)$ are *M*-tuple and *J*-tuple vectors, respectively, with $\alpha \geq 0$.

According to this new definition of weak disposability, we have applied a fixed reduction of α to each input/output measure. Without loss of generality, we assume that this reduction is fixed for each desirable and undesirable output. This assumption will be relaxed later. This reduction decreases output of the undesirable products, *w*, by reducing the activity level in an additive manner.

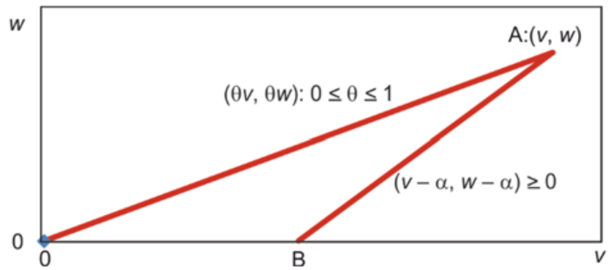

Fig. 1. Two definitions of weak disposability

In Figure 1, we illustrate the above two definitions of weak disposability. Suppose that firm $A:(v, w)$ belongs to the technology set (without loss of generality, we suppose that $v > w$). Based on definition 1, all of the points on the line segment *OA* belong to the production technology set and, as we can see, $v = 0$ if and only if $w = 0$. However, the new definition of weak disposability states that the line segment *AB* belongs to the technology set. At production possibility *B*, we can produce a positive desirable output without producing any undesirable output.

Clearly, the observed levels of desirable and undesirable outputs affect the production technology set. This is illustrated in Fig. 2 for the case of one desirable output and one undesirable output. Here, we have two firms *A* and *B*. In firm *A*, the level of the desirable output *v* is less than the level of the undesirable output *w*, and under the new definition of weak disposability, the line segment *AC* belongs to the technology set. On the other hand, in the case of firm *B*, $v > w$ and based on the assumption of weak disposability, *BD* belongs to the technology set. An interesting point is that by taking the new definition of weak disposability into consideration, we have two different unobserved firms corresponding to the points *C* and *D* in the technology set. At *C*, we have $w > 0$, with zero desirable output and at *D*, $v > 0$ and $w = 0$. The production possibility *D* is feasible, which means that by consuming a certain amount of input, we can produce a positive amount of desirable output without producing any undesirable output. On the other hand, the production possibility C corresponds to a situation where a positive amount of undesirable output must be produced before any desirable output is produced.

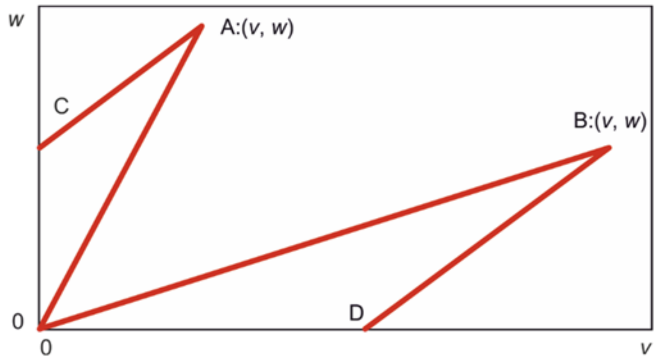

Fig. 2. Weak disposability in two different cases

In what follows, an axiomatic foundation is introduced to construct a new production technology set under the new definition of weak disposability. Our production technology set satisfies the following axioms:

A1. Inclusion of each observation: $(x^k, v^k, w^k) \in T$ for all $k = 1, 2, ..., K$.

A2. Convexity: T is a closed and convex set.

A3. Free disposability: $(x, v, w) \in T$ and $x' \geq x$, $v' \leq v$, $w' = w \Rightarrow (x', v', w') \in T$. (i.e., having more inputs, we can always produce any level of desirable output up to the present level for a given level of undesirable output)

A4. Weak disposability: $(x, v, w) \in T \;\Rightarrow\; 0 \leq (x, v - \alpha_M, w - \alpha_J) \in T$ in which $\alpha_M = (\alpha, \alpha, ..., \alpha) \in \mathbb{R}_+^M$ and $\alpha_J = (\alpha, \alpha, ..., \alpha) \in \mathbb{R}_+^J$.

A5. Minimal extrapolation: T is the smallest set that satisfies the above conditions.

Now, an algebraic representation of the technology set T satisfying the axioms A1–A5 is given.

Theorem 1. The unique production technology set T that satisfies the axioms A1–A5 in a variable returns to scale environment is defined by:

$$T = \{(x, v, w): \; v_m \leq \sum_{k=1}^{K} z^k (v_m^k - \alpha_m), \quad m = 1, 2, ..., M$$

$$w_j = \sum_{k=1}^{K} z^k (w_j^k - \alpha_j), \quad j = 1, 2, ..., J$$

$$x_n \geq \sum_{k=1}^{K} z^k x_n^k, \quad n = 1, 2, ..., N$$

$$v_m^k \geq \alpha_m, \quad m = 1, 2, ..., M$$

$$\sum_{k=1}^{K} z^k = 1$$

$$w_j^k \geq \alpha_j, \quad j = 1, 2, ..., J, \quad z^k \geq 0, \quad k = 1, 2, ..., K\} \tag{1}$$

in which $\alpha_m = \alpha$ is the mth component and $\alpha_j = \alpha$ is the jth component of vectors α_M and α_J, respectively, and $z^k \geq 0$, $k = 1, 2, ..., K$ are intensity variables used to connect inputs and outputs by convex combination.

Proof. T is a nonempty set and clearly it satisfies A1–A4. To show that T is the minimal set, assume that T' satisfies A1–A4. We need to show $(x, v, w) \in T$ implies that $(x, v, w) \in T'$. Consider the following representation of the set of vectors $(x, v, w) \in T$ based on the vector $z = (z^1, z^2, ..., z^K)$.

$$x \geq \sum_{k=1}^{K} z^k x^k, \quad v \leq \sum_{k=1}^{K} z^k (v^k - \alpha_M)$$

$$w = \sum_{k=1}^{K} z^k (w^k - \alpha_J), \quad \sum_{k=1}^{K} z^k = 1$$

$$v^k \geq \alpha_M, \quad k = 1, ..., K, \quad w^k \geq \alpha_J, \quad k = 1, ..., K$$

Such a representation exists, since $T \neq \varnothing$. For the vector $z = (z^1, z^2, ..., z^K)$ from this representation, define

$$\begin{pmatrix} x_z \\ v_z \\ w_z \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{K} z^k x^k \\ \sum_{k=1}^{K} z^k (v^k - \alpha_M) \\ \sum_{k=1}^{K} z^k (w^k - \alpha_J) \end{pmatrix}$$

It is clear that $\left(\sum_{k=1}^{K} z^k x^k, \sum_{k=1}^{K} z^k (v^k - \alpha_M), \sum_{k=1}^{K} z^k (w^k - \alpha_J) \right) \in T'$, and this set of vectors dominates (x, v, w) in the Pareto sense. Hence, we conclude that $(x, v, w) \in T'$. This completes the proof. ■

Since this representation involves the multiplication of pairs of variables, the above technology set T is not in linear form. In what follows, we convert T into a linear form.

Let $\lambda^k = \alpha z^k$ and $\mu^k = (1-\alpha)\, z^k$. Clearly $z^k = \mu^k + \lambda^k$ with $\lambda^k \geq 0$ and since α is not bounded, μ^k is unrestricted in sign. Using these changes of variables, the production technology set T can be rewritten as follows:

$$\begin{aligned} T = \{(x, v, w):\ & v_m \leq \sum_{k=1}^{K} v_m^k (\mu^k + \lambda^k) - \sum_{k=1}^{K} \lambda^k, \quad m = 1, 2, ..., M \\ & w_j = \sum_{k=1}^{K} w_j^k (\mu^k + \lambda^k) - \sum_{k=1}^{K} \lambda^k, \ j = 1, 2, ..., J \\ & x_n \geq \sum_{k=1}^{K} x_n^k (\mu^k + \lambda^k), \ n = 1, 2, ..., N, \ \sum_{k=1}^{K} (\mu^k + \lambda^k) = 1 \\ & v_m^k (\mu^k + \lambda^k) \geq \lambda^k, \ m = 1, 2, ..., M, \ k = 1, 2, ..., K \\ & w_j^k (\mu^k + \lambda^k) \geq \lambda^k, \ m = 1, 2, ..., M, \ k = 1, 2, ..., K \\ & \lambda^k \geq 0, \ k = 1, 2, ..., K\} \end{aligned} \tag{2}$$

This technology set is linear with respect to the variables λ and μ. Using the definition of T, we can interpret the weights $\lambda = (\lambda^1, \lambda^2, ..., \lambda^K)$ and $\mu = (\mu^1, \mu^2, ..., \mu^K)$ as follows: the weights z are decomposed into the sum of two variables μ and λ. This correspondingly decomposes (v, w) into two parts as $(v, w) = (v^\mu, w^\mu) + (v^\lambda, w^\lambda)$. The part (v^μ, w^μ) corresponds to optimal output based on input and (v^λ, w^λ) is those outputs that should be reduced by reducing the activity level. So, μ represents the weights of inputs that are used actively in producing optimal outputs and λ corresponds to the weights of inputs that are used in producing surplus pollutants.

Let us illustrate the new technology set by a simple example consisting of three DMUs, A, B and C, with one desirable output, one undesirable output and one input. The input/output data are summarized in Table 1. Figure 3 illustrates the output set graphically. The horizontal axis shows the levels of the undesirable outputs and the vertical axis corresponds to the levels of desirable outputs. The production set is the hexangular set O2ACB4.

Table 1. The data set for a simple example

DMU	v	w	x
A	3	1	1
B	1.5	4	1
C	3.5	3.5	1

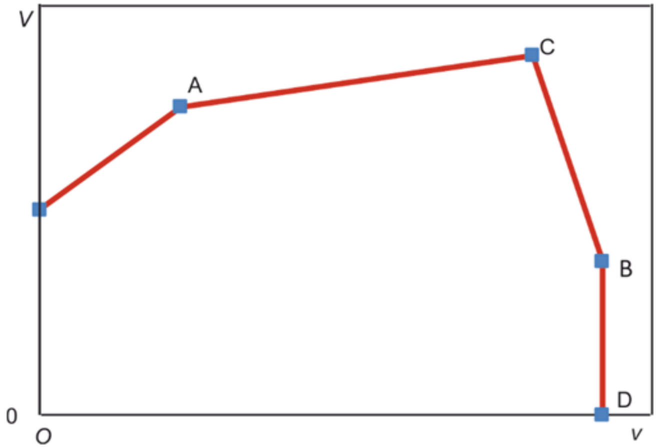

Fig. 3. Output set under the new definition of weak disposability

As the technology set given by (2) shows, the right-hand-side of the constraints are independent of the decision variables. This allows us to use various efficiency measures. Now, we define the directional distance function on T by introducing the reference vector $(v, w, x) = (v^o, w^o, x^o)$. If we want to measure the relative efficiency of firm o in terms of limiting inputs and undesirable outputs and maximising desirable output, we can solve the following linear programming problem:

$$\rho^* = \max \theta$$

s.t.

$$v_m^o + \theta d_v \le \sum_{k=1}^{K} v_m^k (\mu^k + \lambda^k) - \sum_{k=1}^{K} \lambda^k, \quad m = 1, 2, ..., M$$

$$w_j^o - \theta d_w = \sum_{k=1}^{K} w_j^k (\mu^k + \lambda^k) - \sum_{k=1}^{K} \lambda^k, \quad j = 1, 2, ..., J$$

$$x_n^o - \theta d_x \ge \sum_{k=1}^{K} x_n^k (\mu^k + \lambda^k), \quad n = 1, 2, ..., N$$

$$\sum_{k=1}^{K} x_n^k (\mu^k + \lambda^k) = 1$$

$$v_m^k (\mu^k + \lambda^k) \geq \lambda^k, \quad m = 1, 2, ..., M, \quad k = 1, 2, ..., K$$

$$w_j^k (\mu^k + \lambda^k) \geq \lambda^k, \quad m = 1, 2, ..., M, \quad k = 1, 2, ..., K$$

$$\lambda^k \geq 0, \quad k = 1, 2, ..., K \tag{3}$$

where $(d_x,\ d_v,\ d_w)$ is the direction vector defining how $(x^o,\ v^o,\ w^o)$ is projected onto the boundary of T by simultaneously increasing the level of good outputs in the direction d_v and decreasing the level of bad outputs and inputs in the directions d_w and d_x, respectively. θ is the efficiency value measured by the directional distance function associated with the point $(x^o,\ v^o,\ w^o)$.

DMU_o is said to be efficient if and only if the corresponding optimal value of the objective function is equal to one.

The definition of this new production technology set applies a fixed reduction of α to each output. If we apply a reduction of α^k to each DMU_k, the linearized production technology set would be equal to the new technology set (2).

3. Real application

After formulating our theoretical framework, we apply the proposed approach to a real data set covering the period from 1985 to 1998 and consisting of 92 coal-fired power plants with three inputs, one desirable output and two undesirable outputs. The source of these data is the US DOE's EIA-767 survey. The inputs are capital stock, the number of employees and heat content. The two undesirable outputs in this process are sulphur dioxide SO_2 and nitrogen dioxide NO_2. The single desirable product is net electricity generation (in KWh). In what follows, we briefly introduce the input and output variables:

- capital stock: total amount of a plant's capital, represented by the value of its issued common and preferred stock,
- number of employees: the number of employees that work in each power plant,
- heat content: the heat generated by the coal, oil and natural gas consumed at each plant,
- Net electricity generation: the amount of electricity generated by a power plant that is transmitted and distributed for consumer use.
- sulfur dioxide SO_2: is one of a group of gases that is harmful to human health,
- nitrogen dioxide NO_2: is one of a group of gases that is harmful to human health.

These data have been used in several studies [5–9, 12]. Table 2 shows summary statistics for the inputs and outputs of these 92 power plants. Note that the three different variables describing heat in the last three rows have been combined together into a single variable, which is used as one of the three inputs.

Table 2. Summary statistics for 92 coal-fired power plants

Parameter	Mean	Sample st. dev.	Maximum	Minimum
Electricity, million kWh	4686.5	166.6	18212.1	4065.3
SO_2, short ton	40745.2	1293.2	252344.6	48244.8
NO_x, short ton	17494.0	423.1	72524.1	16190.1
Capital stock, million $ in 1973	240.0	39.4	750.0	146.4
Employees	185.2	39.0	535.0	110.9
Heat content of coal, billion Btu	46936.3	1869.3	173436.8	39852.6
Heat content of oil, billion Btu	91.5	0.0	618.9	112.7
Heat content of gas, billion Btu	76.5	0.0	2083.0	275.5

Table 3. Descriptive results based on different reference vectors

Directions	Results	New weak disposability	Kuosmanen and Matin [12]
$(v, w, x) = (0, w_0, 0)$	No. of efficient units	15	26
	Mean efficiency	0.8064	0.2718
	STD	0.3585	0.2337
	RSM	0.8825	0.3576
$(v, w, x) = (v_0, 0, 0)$	No. of efficient units	32	36
	Mean efficiency	0.1583	0.1261
	STD	0.1617	0.1436
	RSM	0.2958	0.1905
$(v, w, x) = (0, 0, x_0)$	No. of efficient units	32	36
	Mean efficiency	0.1021	0.0934
	STD	0.0991	0.0994
	RSM	0.1423	0.1360
$(v, w, x) = (v_0, w_0, 0)$	No. of efficient units	29	34
	Mean efficiency	0.1888	0.1106
	STD	0.1802	0.1222
	RSM	0.2610	0.1643
$(v, w, x) = (v_0, w_0, x_0)$	No. of efficient units	32	36
	Mean efficiency	0.0665	0.0545
	STD	0.0660	0.0605
	RSM	0.0937	0.0812

Two different models have been applied to this data set. Based on the view that the correct and complete approach to modelling undesirable outputs in production analysis is presented by Kuosmanen [10], we first applied the approach that takes the classical definition of weak disposability into consideration and then applied our proposed model, given by (3). Five different reference points have been applied to this example. These reference points are:

$$(v, w, x) = (0, w_o, 0),\ (v, w, x) = (v_o, 0, 0), (v, w, x) = (0, 0, x_o)$$
$$(v, w, x) = (v_o, w_o, 0), \text{ and } (v, w, x) = (v_o, w_o, x_o)$$

A statistical description of the results are summarized in Table 3 and (Figs. 4–8). The second column shows the results based on our proposed model. As we can see, the minimum number of efficient plants is fifteen, which occurs based on the reference vector $(v, w, x) = (0, w_o, 0)$. Based on our approach, the maximum mean efficiency is 0.8064 with respect to the reference vector $(0, w_o, 0)$, while the minimum mean efficiency occurs with respect to the reference vector $(v, w, x) = (v_o, w_o, x_o)$. The results of Kuosmanen [10] are also listed in the fourth column of Table 3. Based on this approach, the minimum number of efficient plants is 26, again with respect to the reference vector $(v, w, x) = (0, w_o, 0)$.

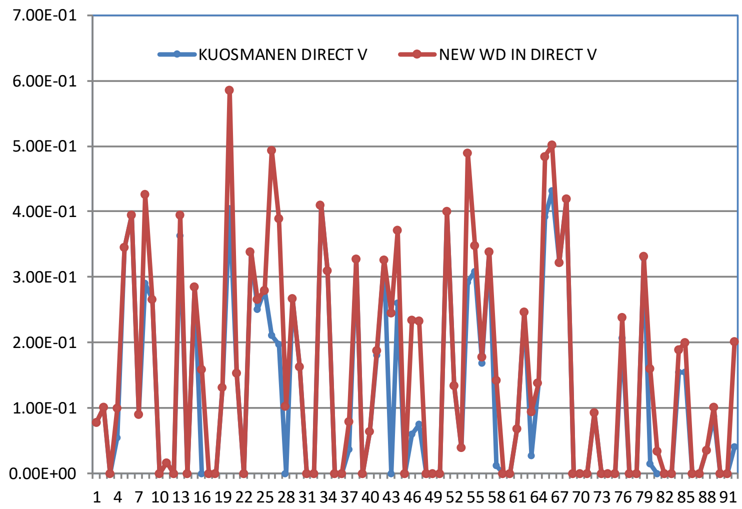

Fig. 4. Efficiencies with respect to reference vector $(v, w, x) = (v^o, 0, 0)$

By comparing the mean efficiencies according to these two approaches, we have found that the maximum and minimum differences between corresponding mean efficiencies are 0.5346 with respect to the reference vector $(0, w_o, 0)$ and 0.0087 with respect to the reference vector $(0, 0, x_o)$, respectively.

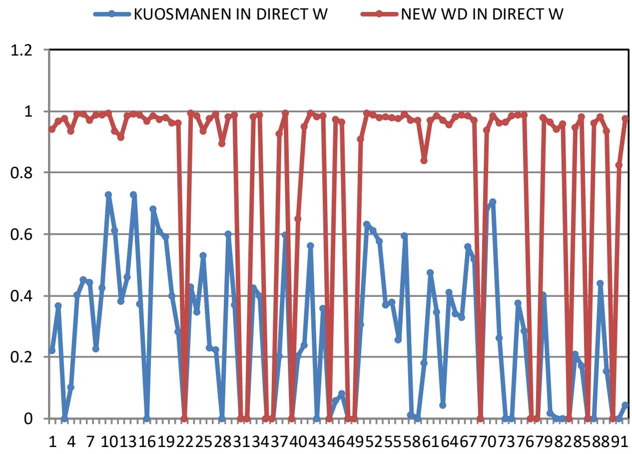

Fig. 5. Efficiencies with respect to reference vector $(v, w, x) = (0, w^o, 0)$

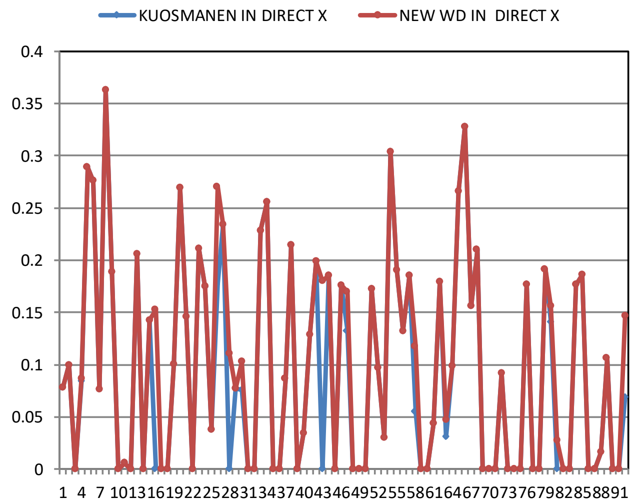

Fig. 6. Efficiencies with reference vector $(v, w, x) = (0, 0, x^o)$

We can see that although the number of efficient firms according to our new approach is less than that according to Kuosmanen [10] with respect to each of the five reference vectors, there is no large difference between the numbers of efficient firms according to the two approaches. We have found that plants which are efficient according to our proposed approach are also efficient according to Kuosmanen [10] but the converse is not necessarily true. In conclusion, the power of the proposed approach to discriminate between firms seems to be better than the power of the existing approaches.

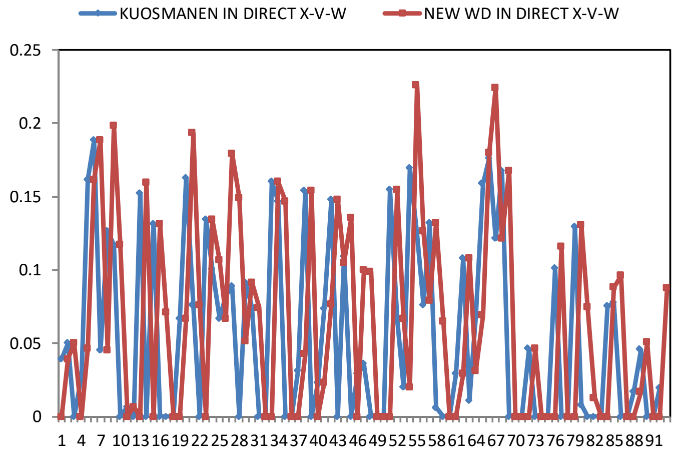

Fig. 7. Efficiencies with respect to reference vector $(v, w, x) = (v^o, w^o, x^o)$

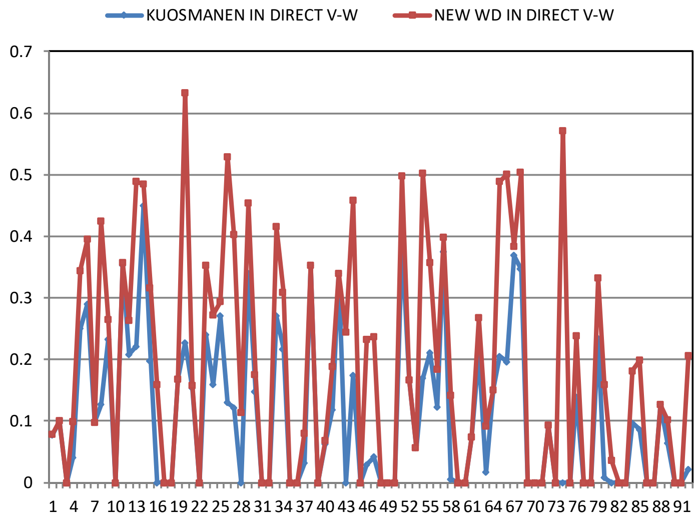

Fig. 8. Efficiencies with respect to reference vector $(v, w, x) = (v^o, w^o, 0)$

We have also calculated the root square means (RSM) according to these two approaches and the maximum and minimum RSM differences are 0.5249 based on the reference vector $(0, w_o, 0)$ and 0.0063 based on the reference vector $(0, 0, x_o)$. In conclusion, the results based on the reference vector $(0, w_o, 0)$ are highly dependent on the approach used, whereas, based on the reference vector $(0, 0, x_o)$, the two approaches give very similar results.

It should be pointed out that these comparisons do not clearly show that our proposed approach is better than the often used approach of Kuosmanen [10] – both assume weak disposability – and we do not claim that there is a weakness in their approach. We believe that the technology set defined by Kuosmanen [10] and Kuosmanen and Matin [12] is the complete and correct technology set. Hence, we have compared these two approaches to show the similarity of the results obtained.

4. Conclusions

A classical definition of weak disposability of Shephard [16] has been used to model undesirable factors in production processes as outputs. To this end, firms abate undesirable outputs by proportionally decreasing their activity level. This paper proposes a new definition of weak disposability in an additive form and by taking this new definition into consideration, an axiomatic foundation is introduced to construct a new production technology set. The proposed assumption of weak disposability preserves the linearity of the new production technology set and is fundamentally different from those that have been proposed in previous studies. The approach proposed in this paper is applicable when zero undesirable (desirable) outputs do not necessarily require zero desirable (undesirable) outputs. The applicability of this model is illustrated by a real life case of 92 firms with two undesirable outputs. The model proposed in this paper assumes that the production process is a simple single-stage process and the input/output data set is deterministic and crisp. Future studies could extend this approach to a network-structured production process for which the data set stochastic.

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