NAUKI O FINANSACH FINANCIAL SCIENCES 4(21) • 2014
ISSN 2080-5993

Jerzy Tymiński
The Jan Kochanowski University in Kielce
e-mail: tymmar@poczta.onet.pl

A CONCEPT OF USING A DYNAMIC PROGRAMMING METHOD TO OPTIMIZE AN INVESTMENT PORTFOLIO ALLOWING FOR ASYMMETRIC RATES OF RETURN AND A MINIMUM RISK PORTFOLIO

Summary: Investment management on the capital market is a complex and multifarious process and the accuracy of decisions is an indispensable condition that an investor needs to fulfill if the expected economic results are to be achieved. The paper presents the concept of the optimization of investment portfolio on the capital market of shares. The maximum value of portfolio quality measure was used as an optimization criterion. It is expressed by the index of variability R/σ of the rate of return for each share in the portfolio. The cumulation of values of R/σ index in the successive years of the investigated period allowed for an econometric estimation of the continuous functions and their maximum. The indexes of asymmetry of rate of return for particular shares in the portfolio were introduced into the functions, which enabled to increase the efficiency of the selection of shares for the portfolio. This, in turn, allowed to achieve the optimum structure of shares in the portfolio.

Keywords: optimization, dynamic programming, asymmetry, risk.

DOI: 10.15611/nof.2014.4.04

1. Introduction

Modern theories of economics and finance which examine market processes including capital investments have been developing very dynamically. The world literature of the subject is abundant with papers devoted to the construction of portfolio models and their optimization [Lakner 2006]. In the Polish literature the problem of investment on the capital market is reflected in numerous publications and research studies conducted by eminent representatives of this field of science, e.g. K. Jajuga, W. Tarczyński, and M. Kolupa to name but a few.

The aim of an investor active on the capital market is to construct an optimal investment portfolio, i.e. a portfolio of shares of high effectiveness characterized by an optimum between income and risk and including the function of the investor's profit. The application of multi-criteria principles of portfolio construction is required to accomplish this aim.

The research problem of the paper consists in constructing a two- criterion portfolio which enables to maximize income while simultaneously minimizing risk. Moreover, in the presented concept, the author takes into account the asymmetry of the rates of return which increased the effectiveness of the market portfolio.
The subject is further investigated with the use of the dynamic programming method and statistical- econometric tools.

The main aim of the present paper is to have a voice in the discussions on the problem of the construction and optimization of a portfolio of shares on the capital market. The proposed approach is an alternative model proposal which widens the range of possibilities of the evaluation accuracy of the investor's decision on the selection of the model of investment portfolio.

The process of building a bi-criterion portfolio is reflected in the concept of optimizing an investment portfolio discussed in this paper. The concept is justified by the following circumstances [Tymiński, Zawiślak 2008].

- Information on stock values, usually available (ex post) for a long time horizon, does not always fully show the trends of the rates of return. The calculated risk may also differ from its actual level in the future [Haugen 1996].
- Estimating the probability that a pessimistic, moderate or optimistic scenario for a financial instrument portfolio will materialise, we frequently have to make assessments that are subjective to a lower or higher degree.
- The historical data also include negative rates of return. When the portfolios of financial instruments (stocks) are built using the traditional methods and then optimized, the negative rates of return (especially those occurring in the last periods) may affect the stock portfolio structure even for the positive values of the expected rates of return in the ex-post observation.
- The life cycles of the products on which the selected companies base their stocks are usually much longer than for other marketed goods.

2. The stock portfolio optimization model

We assume that the criterion (purpose) function is constructed based on the maximization of the "profit function" that expresses the cumulated converse of the periodical (monthly) stock variability coefficients. Hence, the cumulated purpose function is modified with partial (monthly) variability coefficients and can take either a minimum (in the pure form, for σ/R – the standard deviation to the rate of return ratio or a maximum for R/σ). We introduce the modified variability coefficient as a formula:

$$Wz_i(t) = \frac{\sigma_{pi}}{E(R_i(t))}, \tag{1}$$

where i – company numbers equivalent to the portfolio numbers (1).

The modification consists of taking σ_{pi} as the standard deviation of the entire portfolio for the i^{th} company Sharpe's model is applied) and $E(R_i(t))$ – the expected rate of return for the i^{th} company in the i^{th} period.

In this case, variance has the form $\sigma_{pi}^2 = \beta^2\sigma_M^2 + \sigma_{e_i}^2$. In the presented concept we will use the inverse of the variability coefficient $OWz(t)$.

The other model assumptions are: the criterion function is an estimated trend function and the companies are selected based on the values of the determination coefficient (while maintaining the significance of the trend model's structural parameters).

The optimization model for the ultimate stock portfolio is of the form:

$$\max\left(\sum_{i=1}^{n} tx_i \frac{1}{Wz_i}\right). \tag{2}$$

The boundary condition is the sum of the stocks of the considered (selected) companies. $\sum_{i=1}^{n} tx_i = 1$ where n is the number of companies included in the portfolio model undergoing optimization assuming additionally that the sum of the company stocks' portions in the portfolio respect $tx_i \in \langle 0,1 \rangle$.

The optimization model solving process is divided into several steps.

First, companies are selected using a measure of reliability[1]. Statistically, the measure of reliability (an approximate value) can be expressed as:

$$\hat{R}(t) = \frac{\text{the number of products (the positive rates of return)}}{\begin{array}{c}\text{the number of products in time } t = 0 \text{ (here understood as the}\\ \text{sum of the initial rates of return in all observed periods).}\end{array}} \tag{3}$$

According to the presented concept, the measure is [Tymiński 2013, p. 118]:

$$\hat{R} = \frac{\text{the sum of all positive rates of return (\%)}}{\text{the sum of modules of the positive rates of return (\%)}} \tag{3a}$$

The stock portfolio optimization model was solved with the dynamic programming method, using a specific example of observations (data) for companies listed on the stock exchange (see Table 1).

[1] In classical terms, the reliability measure is given by Wiener's formula of the form $R(t) = e^{-\int_0^t \lambda(t)dt}$, where $\lambda(t)$ – the intensity of the observed "inadequacies" of the rate of return (a negative start).

3. The dynamic programming method

In the business world we are frequently forced to make decisions when means and resources are tight. The problem that we have to solve then concerns the rational allocation of what is available, i.e. we have to select an operational variant that will perform best against the specific criterion. This is a typical area where the decisions to be made must be optimal in terms of the accepted criteria. In the decision-making processes concerning the stock market (they frequently accompany planning and the forecasting of actions) dynamic programming algorithms may turn out to be useful.

Dynamic programming is a mathematical approach making use of the so-called Bellman's principle of optimality. This understanding of operational optimality has the property that, irrespective of the initial state and of the initial decisions the successive decisions, have to be optimal because of the situation caused by the first decision. In the reverse situation, first the optimal values for the last state N are determined and then the process moves backwards to find the optimal solution to the state $N-1$ (the last but one). Step by step, the process leads us back to the first stage. The applied operational procedure (the solution) arises from that principle [Bellman 1965].

A simple optimization model as prescribed by the dynamic programming convention is therefore of the form:

$$\max Z(w_1, w_2, \ldots, w_n) = g(w_1) + g(w_2) + \ldots + g(w_n) \tag{4}$$

under the constraints $w_1 + w_2 + \ldots + w_n = k$ where $w_1, w_2, \ldots, w_n$ are decision variables standing for the quantities of resources that can be allocated to carrying out particular tasks and $g(w_1), g(w_2), \ldots, g(w_n)$ are the profit functions for the resources invested in the tasks.

4. The portfolio optimization concept utilising the dynamic programming method

Applying a dynamic programming approach enhanced by a reliability analysis that uses the trends in the selected stocks' rates of return that occur across the available set of observations (for several listed companies) may be rational. According to the equation, this decision-making problem requires determining the optimal distribution of k resources among n projects, so as to maximise the expected effect of an action (a financial result, production output, firm's value, etc.).

Table 1. Rate of return

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	Expected
AMC	6.11	1.40	0.47	3.32	6.86	–3.16	–27.55	–10.00	–9.82	3.64	–2.87
ELE	–59.13	73.22	34.69	–3.12	9.09	–13.33	–11.76	–23.68	–27.66	3.33	–1.84
IRE	1.56	9.52	0.88	12.39	6.06	–13.99	7.55	–14.29	–15.18	–0.78	–0.63
KLR	–1.56	10.26	9.71	0.63	17.20	23.85	–7.07	20.75	4.17	9.94	8.79
MCI	4.70	3.10	–1.02	6.47	–1.85	–7.25	1.55	–15.88	10.00	2.06	0.19
NET	12.50	–2.04	–10.00	–5.45	5.17	16.36	0.00	–2.17	4.26	6.67	2.53
PKO	4.18	2.56	8.42	–3.16	0.28	6.70	–9.28	7.95	10.12	3.31	3.11
SKA	2.98	4.44	–1.43	3.87	6.96	7.48	6.38	–13.64	–2.00	–9.02	0.60
TFM	2.70	22.78	–6.24	2.93	0.00	3.72	–0.72	6.95	9.79	–1.43	4.05
WST	6.67	–3.93	–3.09	6.67	–6.43	–8.72	–3.70	–1.19	–24.12	–2.13	–4.00
WIG	7.23	4.09	2.95	5.24	–19.60	23.91	–4.51	6.99	3.82	6.67	3.68

Source: developed by the authors using data derived from the [Internet 1].

It is practical to choose the companies using the reliability coefficient $\hat{R}(10)$. Because for the companies *AMC*; *ELE*, *IRE* and *WST*, the expected rates of return are negative, they will be omitted from the reliability assessment. Ultimately, the reliability assessment procedure provided the following companies (formula 3): *KLR*($\hat{R}(10)_{KLR}$= 0.9851), *TFM* ($\hat{R}(10)_{TFM}$= 0.8535) and *PKO* ($\hat{R}(10)_{PKO}$= 0.7777).

At the next stage, according to the formula (1), the modified variability coefficients are constructed for the selected companies (using 10 monthly periods). Table 2 presents the standard deviations for the portfolio of selected company's stocks.

Table 2. Standard deviations for the portfolio

KLR	0.0996	TFM	0.0795	PKO	0.0592

Source: calculated by the authors.

The next stage comes down to calculating the converse of the modified variability coefficients $OWz_i(t)$. Finally we can calculate the trend function as:

$$F_{Owz_{KLR}}(t) = -9.4806t^3 + 15.5224t^2 + 3.1905t - 0.5179,$$

$$F_{Owz_{FTM}}(t) = 20.9866t^3 - 35.4604t^2 + 21.3486t - 0.8738,$$

$$F_{Owz_{PKP}}(t) = 23.9733t - 36.0019t^2 + 18.4307t - 0.8994.$$

The estimated models have good estimation quality (R^2 = 85% and significant structural parameters estimated by means of *t*-Student statistics).

5. Asymmetry coefficients in the stock portfolio optimization processes

The procedures used to construct a financial instrument (mainly stocks) portfolio usually assume that the rates of return have a normal distribution. However, we are usually uncertain whether the given distribution is really normal[2]. It can show an asymmetry that is important for the portfolio optimization processes. It is vital for the investor to know what direction asymmetry may take (i.e. the deviation from the model). The right-sided asymmetry is more favourable as it offers a stronger probability of earning a higher rate of return than average. We can calculate the asymmetry using the formula:

$$A_i = \frac{n}{(n-1)(n-2)} \sum_{j=1}^{n} \frac{(x_j - \overline{x})}{\sigma_{pi}}, \tag{5}$$

where: n – the number of the (monthly) observations of the rates of return (in our case there are 10 of them), x_j – j-th – the monthly rate of return for the stocks in the i-th portfolio (i = *KLR*, *TFM*, *PKO*), $\overline{x}$ – the average monthly rate of return. The standard deviation of the portfolio's rate of return. The standard deviations are determined using Sharpe's approach. The coefficient A_i is calculated for the particular selected companies. A_{KIR} = –0.001. This result indicates a slightly left-sided asymmetry. For the other companies, the results are as follows: A_{TFM} = 1.519 and A_{PKO} = 0.130 . Both the coefficients show right-sided asymmetries (for *TFM* it is even high). Finally, the calculated coefficients are introduced to the estimated functions $F_{Owz_i}(t)$ as the product [Tymiński 2013, p. 121]:

$$F^*_{Owz_i}(t) = F_{Owz_i}(t) \cdot (1 + A_i). \tag{6}$$

The next step is to recalculate the trend function values for the chosen companies taking into account its asymmetry.

Generally, the dynamic programming method has a combinatory form, because all resources have to be distributed among the three companies (in accordance with the values of function $F^*_{Owz_i}(t)$).

Let us introduce the following notations: $F_{1,2}(t_x)$ – the yield for the optimal distribution of resources among companies 1 and 2; $F_{2,3}(t_x)$ – the yield for the optimal distribution of resources among companies 1, 2 and 3.

The algorithm for a solution involving the dynamic programming method. In the first step we calculate $F_{1,2}(t_x)$ for $t_x \in <0,1>$. At the second stage of the optimization

---

[2] In order to determine whether a distribution is normal the measures of concentration K are calculated. For $K = 0$ the distribution is normal (for $K \neq 0$ it is convex). Then the asymmetry coefficient should be calculated.

process, the optimal $F_{1,2,3}(t_x)$ values are being looked for. Hence, for $F_{1,2,3}(t_x) = \max(F_{1,2}(t_x) + F_3(t_x))$.

The optimization results (i.e. of maximising the measure of performance $F^*_{Owz_i}(t)$) are presented in Table 3.

The maximal yield, in terms of the values of the measures $F^*_{Owz_i}(t)$, is obtained for the following structure of the companies' shares in the portfolio: *KLR*'s share should be zero (0.0), *TFM*'s should be one (1.0); and for *PKO* also zero (0.0). For such a distribution the portfolio's expected rate of return will be obtained:

$$R_p = R_{TFM} = 4.05\%$$

To estimate portfolio risk we use a measure of the form $F^*_{Owz_i}(t)$. Therefore, the optimal value of the indicator will be: $F^*_{Owz_i}(t) = \dfrac{4.05}{0.0795}(1+1.519) = 128.33$. Ultimately, the original indicator of quality (before transformation in the optimization process), i.e. the variability coefficient, will have the value $Wz = 0.779\%$[3].

Table 3. Optimization results

Distribution of "resources"	Yield per resources allocated to the companies $F_{1,2,3}(tx)$	Optimal strategy of allocating resources to the companies	
		KLR and *TFM*	*KLR*, *TFM*, *PKO*
0	0.000	0.0; 0.0	0.0; 0.0; 0.0
0,1	2.337	0; 0.1	0; 0.1; 0
0,2	5.407	0; 0.2	0; 0.2; 0
0,3	7.323	0; 0.3	0; 0.3; 0
0,4	8.405	0; 0.4	0; 0.4; 0
0,5	8.968	0; 0.5	0; 05; 0
0,6	9.331	0; 0.6	0; 06; 0
0,7	9.983	0.3; 0.4	0; 0.6; 0.1
0,8	11.092	0.5; 0.3	0; 0.5; 0.3
0,9	12.255	0.6; 0.3	0; 0.9; 0;
1,0	15.122	0; 1.0	0; 1.0; 0

Source: calculated by the authors.

However the optimization process has to go on if we want to account for short sales[4]. Then the maximal value of the combinations $F^*_{Owz_i}(t) = F_{Owz_i}(t)\cdot(1+A_i)$ for

[3] This value of the standard deviation was obtained by optimizing the portfolio according to the investment portfolio optimization concept presented in the article. The *TFM* value is lower than the predicted values, because it expresses the probability that the forecast $R_{p(TFM)}$ will be true (this results from the value of the distribution asymmetry of the rate of return for *TFM*).

[4] The problem was discussed in [Tymiński 2011].

$t_x > 1$ needs to be determined for *KLR*, *TFM* and *PKO*. At the same time, $\min(\sigma_{pi})$ has to be estimated.

A different procedure is applied to identify minimal risk and more precisely the MVP (the minimum variance portfolio). It uses formulas for defining the weights for the portfolio stocks (e.g. *PKO* and *TFM*):

$$w_1 = (\delta_2^2 - \delta_1\delta_2\xi_{12}) / (\delta_1^2 + \delta_2^2 - 2\delta_1\delta_2\xi_{12} \text{ [Jajuga, Jajuga 1998, p. 131]}, \quad (7)$$

where $\xi_{1,2}$ denotes the correlation coefficients.

In this procedure, the stock with the lowest value of the measure $PWz_i(t)(1+A)$ *OWA* is selected. This is the *PKO* stock whose rate of return is 3.11% and the standard deviation is δ_{PKO} =1.672%.Therefore, for the *TFM* and *PKO* stocks, we have:

- the rates of return: 4.05% and 3.11%, respectively;
- the standard deviations: 0.779% and 1.672%, respectively;
- the correlation coefficients: $\xi_{TFM,PKO} = 0.149$.

The values of the relation indicate that the minimal risk portfolio is offered by the non-negative companies' shares in the portfolio. Substituting the values calculated above into the relation (6), we have: $w_{PKO} = 0.136$ and $w_{TFM} = 0.864$.

The ultimate composition of the minimal risk portfolio is as follows:

$$R_p^* = 0.864 \cdot 4.05\% + 0.136 \cdot 3.11\% = 3.92\%.$$

$$\delta_p^* = 0.864 \cdot 0.779\% + 0.136 \cdot 1.672\% = 0.90\%$$

Even if the optimization procedure proposed in the article yielded a portfolio with short sales, it would not guarantee (with the exception of formula 6) that a minimal risk portfolio would be created. On the other hand, a portfolio optimization procedure using the traditional approach allows assembling a minimal risk portfolio, but it does not ensure a fail-safe choice of stocks for the portfolio, which may raise doubts about its durability, even in the medium-range forecast (e.g. a two-year forecast).

6. Conclusion

The proposed concept of optimizing an investment portfolio builds on dynamic programming and (at the first stage, when the portfolio stocks are being selected) on some elements of reliability theory. Moreover, the concept also provides the options of introducing short sales and particularly of creating a minimal risk portfolio. Another advantage of using the cumulated values of the modified variability coefficients' trend functions is that the approach takes account of the measures' time trends (in the short or even medium-range forecasts) in the optimized investment portfolio.

Using reliability theory to decide about financial instruments to be included in a portfolio makes the optimized portfolio “more durable”. This aspect is important for investors who want their portfolios to be profitable over possibly long periods of time.

References

Bellman R., 1965, *Adaptacyjne procesy sterowania*, PWN, Warszawa.

Haugen R., 1996, *Teoria nowoczesnego inwestowania*, WIG PRESS, Warszawa.

Jajuga K., Jajuga T., 1998, *Inwestycje. Instrumenty finansowe. Ryzyko finansowe*, PWN, Warszawa.

Kryński H., Badach A., 1976, *Zastosowania matematyki do podejmowania decyzji ekonomicznych*, PWE, Warszawa.

Lakner P., 2006, *Portfolio Optimization with Downside Constraints*, “*Mathematical Finance”*, vol. 16, no. 2, New York University.

Tarczyński W., 1997, *Rynki kapitałowe. Metody ilościowe*, Placet, Warszawa.

Tymiński J., 2013, *Ekonomiczne aspekty optymalizacji inwestycji długookresowych*, Warszawa.

Tymiński J., 2011, *Podstawy procesów decyzyjnych na rynku kapitałowym*, WSGK, Kutno.

Tymiński J., Zawiślak R., 2008, *Dwukryterialna koncepcja wyboru instrumentów finansowych dla efektywnej konstrukcji portfela i jego optymalizacja na rynku kapitałowym*, Uniwersytet Szczeciński, Szczecin.

Tymiński J. Zawiślak R., 2009, *Wykorzystanie elementów teorii sterowania w problemie optymalizacji portfela inwestycyjnego na rynku kapitałowym*, Uniwersytet Szczeciński, Szczecin.

Internet

[1] www.rynek.owg.pl.

KONCEPCJA OPTYMALIZACJI PORTFELA INWESTYCYJNEGO METODĄ PROGRAMOWANIA DYNAMICZNEGO Z UWZGLĘDNIENIEM ASYMETRII STÓP ZWROTU ORAZ MINIMALNEGO RYZYKA

Streszczenie: Zarządzanie inwestycjami na rynku kapitałowym jest procesem złożonym i wielopłaszczyznowym, a trafność podejmowanych decyzji jest warunkiem koniecznym skuteczności inwestora w osiąganiu oczekiwanych efektów ekonomicznych. W artykule przedstawiono koncepcję optymalizacji portfela inwestycyjnego na rynku kapitałowym akcji. Jako kryterium optymalizacji zastosowano maksymalną wartość miernika jakości portfela. Wyraża go wskaźnik zmienności R/σ dla stopy zwrotu każdej akcji w portfelu. Skumulowanie wartości wskaźnika R/σ w kolejnych latach okresu badawczego pozwoliło oszacować ekonometrycznie funkcje ciągłe i ich maksimum. Do tych funkcji wprowadzono wskaźniki asymetrii stóp zwrotu poszczególnych akcji portfela, co zwiększyło efektywność doboru akcji do portfela. Oznacza to osiągnięcie optymalnej struktury akcji spółek w portfelu.

Słowa kluczowe: optymalizacja, programowanie dynamiczne, asymetria, ryzyko.

Annex 1. Prices of stocks of selected companies listed on WSE

	Price (PLN)										
	1 June 2005	1 July 2005	1 Aug. 2005	1 Sept. 2005	1 Oct. 2005	1 Nov. 2005	1 Dec. 2005	1 Jan. 2006	1 Feb. 2006	1 March 2006	1 April 2006
APL	3.12	3.10	3.00	2.96	2.89	2.63	3.52	4.02	4.42	6.60	6.15
BDX	45.50	46.90	43.00	43.30	40.00	40.00	36.00	38.00	45.00	45.00	46.80
GRJ	22.40	21.85	22.30	21.30	26.40	27.50	34.70	35.10	37.00	35.70	38.00
GTC	114.30	112.00	122.00	135.00	144.00	136.00	145.00	172.00	203.00	260.50	286.00
INT	10.21	9.70	11.70	11.65	11.15	9.90	18.10	18.90	27.90	28.00	30.20
JTZ	69.50	70.00	74.20	79.00	81.90	74.90	75.00	86.50	77.00	29.10	73.80
KRS	11.20	12.00	10.50	8.45	8.90	8.00	7.60	7.95	8.00	7.00	7.25
PEO	141.90	143.50	150.50	163.50	181.50	157.00	176.00	174.50	173.30	188.50	190.50
PKM	100.35	99.00	111.50	116.00	116.00	123.00	127.50	136.50	140.00	149.00	147.50
RPC	16.90	18.55	20.80	20.30	20.20	21.00	20.80	24.40	25.50	26.00	27.50
SKA	25.65	27.30	24.60	24.50	22.30	23.30	25.20	26.60	28.00	27.60	29.80
WWL	118.75	119.50	120.00	122.50	139.00	136.00	150.00	168.00	171.00	232.00	234.00
WIG	29539.21	28308.71	30408.01	31479.57	33898.06	31937.43	34012.14	35466.39	37423.17	3902590	40199.06

Source: developed by the author based on data derived from the ”PARKIET” gazette.

Rates of return for 10-month periods

	Monthly rates of return on stocks (from 1 June 2005 to 1 March 2006; %)										
	Rate of return r_1	Rate of return r_2	Rate of return r_3	Rate of return r_4	Rate of return r_5	Rate of return r_6	Rate of return r_7	Rate of return r_8	Rate of return r_9	Rate of return r_{10}	Expected rate of return r_i
APL	–0.64	–3.23	–1.33	–2.36	–9.00	33.84	14.20	9.95	49.32	–6.82	8.39
BDX	3.08	–8.32	0.70	–7.62	0.00	–10.00	5.56	18.42	0.00	4.00	0.58
GRJ	–2.46	2.06	–4.48	23.94	4.17	26.18	1.15	5.41	–3.51	6.44	5.89
GTC	–2.01	18.75	1.50	6.67	–5.56	6.62	18.62	18.02	28.33	9.79	10.07
INT	–5.00	20.62	–0.43	–4.29	–11.21	82.83	4.42	47.62	0.36	7.86	14.28
JTZ	0.72	6.00	6.47	3.67	–8.55	0.13	15.33	–10.98	–10.26	6.80	0.93
KRS	7.14	–12.50	–19.52	5.33	–10.11	–5.00	4.61	0.63	–12.50	3.57	–3.84
PEO	1.13	4.88	8.64	11.01	–13.50	12.10	–0.85	–0.69	8.77	1.06	3.25
PKM	–1.35	12.63	4.04	0.00	6.03	3.66	7.06	2.56	6.43	–1.01	4.01
RPC	9.76	12.13	–2.40	–0.49	3.96	–0.95	17.31	4.51	1.96	5.77	5.16
SKA	6.43	–9.89	–0.41	–8.98	4.48	8.15	5.56	5.26	–1.43	7.97	1.72
WWL	0.63	0.42	2.08	12.47	–2.16	10.29	12.00	1.79	35.67	0.86	7.51
WIG	–4.17	7.42	3.52	7.68	–5.78	6.50	4.28	5.52	4.28	3.01	3.23

Source: developed by the author based on data derived from the ”PARKIET” gazette.