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Metody i zastosowania badań operacyjnych

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Wstęp

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Kolejna, XXXIV Ogólnopolska Konferencja Naukowa im. Profesora Władysława Bukietyńskiego, organizowana corocznie przez najważniejsze ośrodki naukowe zajmujące się dziedziną badań operacyjnych, w roku 2015 odbyła się w pięknym, zabytkowym i świeżo odremontowanym zespole pałacowo-parkowym w Łagowie koło Zgorzelca. Konferencję zrealizowaną pod nazwą *Metody i Zastosowania Badań Operacyjnych* przygotowała Katedra Badań Operacyjnych Uniwersytetu Ekonomicznego we Wrocławiu pod kierownictwem dr. hab. Marka Nowińskiego, prof. UE.

Konferencje te mają już długoletnią tradycję – są to coroczne spotkania pracowników nauki specjalizujących się w badaniach operacyjnych. Głównym celem konferencji było, podobnie jak w latach ubiegłych, stworzenie (przede wszystkim dla młodych teoretyków, a także praktyków dyscypliny) forum wymiany myśli na temat najnowszych osiągnięć dotyczących metod ilościowych wykorzystywanych do wspomagania procesów podejmowania decyzji, a także prezentacja nowoczesnych zastosowań badań operacyjnych w różnych dziedzinach gospodarki. Ten cenny dorobek naukowy nie może być zapomniany i jest publikowany po konferencji w postaci przygotowywanego przez organizatorów zeszytu naukowego zawierającego najlepsze referaty na niej zaprezentowane.

W pracach Komitetu Naukowego Konferencji uczestniczyli czołowi przedstawiciele środowisk naukowych z dziedziny badań operacyjnych w Polsce; byli to: prof. Jan B. Gajda (Uniwersytet Łódzki), prof. Stefan Grzesiak (Uniwersytet Szczeciński), prof. Bogumił Kamiński (SGH w Warszawie), prof. Ewa Konarzewska-Gubała (Uniwersytet Ekonomiczny we Wrocławiu), prof. Donata Kopańska-Bródka, prof. Maciej Nowak i prof. Tadeusz Trzaskalik (Uniwersytet Ekonomiczny w Katowicach), prof. Dorota Kuchta (Politechnika Wrocławska), prof. Krzysztof Piasecki (Uniwersytet w Poznaniu) i prof. Józef Stawicki (Uniwersytet Mikołaja Kopernika w Toruniu).

Zakres tematyczny konferencji obejmował teoretyczne i praktyczne zagadnienia dotyczące przede wszystkim:

- modelowania i optymalizacji procesów gospodarczych,
- metod wspomagających proces negocjacji,
- metod oceny efektywności i ryzyka na rynku kapitałowym i ubezpieczeniowym,
- metod ilościowych w transporcie i zarządzaniu zapasami,
- metod wielokryterialnych,
- optymalizacji w zarządzaniu projektami oraz analizy ryzyka decyzyjnego.

W konferencji wzięło udział 43 przedstawicieli różnych środowisk naukowych, licznie reprezentujących krajowe ośrodki akademickie. W trakcie sześciu sesji plenarnych, w tym dwóch sesji równoległych, przedstawiono 27 referatów, których poziom naukowy w przeważającej części był bardzo wysoki. Zaprezentowane referaty, po pozytywnych recenzjach, zostają dziś opublikowane w Pracach Naukowych Uniwersytetu Ekonomicznego we Wrocławiu w postaci artykułów naukowych w specjalnie wydanym zeszycie konferencyjnym.

Przypominając przebieg konferencji, nie można nie wspomnieć o konkursie zorganizowanym dla autorów referatów niebędących samodzielnymi pracownikami nauki. Dotyczył on prezentacji najciekawszego zastosowania badań operacyjnych w praktyce gospodarczej. Komitet Organizacyjny Konferencji powołał kapitułę konkursu, w której skład weszli: prof. Ewa Konarzewska-Gubała – przewodnicząca, prof. Jan Gajda, prof. Stefan Grzesiak i prof. Donata Kopańska-Bródka. Członkowie Komisji Konkursowej oceniali referaty ze względu na:
- innowacyjność, oryginalność metody będącej przedmiotem zastosowania,
- znaczenie zastosowania dla proponowanego obszaru,
- stopień zaawansowania implementacji metody w praktyce.

Spośród 15 referatów zgłoszonych wyróżniono: 1. miejsce: dr Michał Jakubiak i dr hab. Paweł Hanczar (Uniwersytet Ekonomiczny we Wrocławiu), *Optymalizacja tras zbiórki odpadów komunalnych na przykładzie MPO Kraków*; 2. miejsce: mgr Dagmara Piesiewicz i dr hab. Paweł Hanczar (Uniwersytet Ekonomiczny we Wrocławiu), *Logistyka odzysku – optymalizacja przepływów w systemie gospodarki komunalnej*; 3. miejsce: dr Dorota Górecka i dr Małgorzata Szałucka (Uniwersytet Mikołaja Kopernika w Toruniu), *Wybór sposobu wejścia na rynek zagraniczny – wieloaktorska analiza wielokryterialna a podejście oparte na dominacjach stochastycznych*.

Przy okazji prezentowania opracowania poświęconego XXXIV Konferencji *Metody i Zastosowania Badań Operacyjnych* i jej bardzo wartościowego dorobku nie możemy nie podziękować członkom Komitetu Organizacyjnego Konferencji, w którego skład wchodzili młodzi, acz doświadczeni pracownicy Katedry Badań Operacyjnych Uniwersytetu Ekonomicznego we Wrocławiu: dr Piotr Peternek (sekretarz), dr hab. Marek Kośny, dr Grzegorz Tarczyński oraz mgr Monika Stańczyk (biuro konferencji). Zapewnili oni w sposób profesjonalny sprawne przygotowanie i przeprowadzenie całego przedsięwzięcia oraz zadbali o sprawy administracyjne związane z realizacją konferencji, a także byli odpowiedzialni za dopilnowanie procesu gromadzenia i redakcji naukowych materiałów pokonferencyjnych, które mamy okazję Państwu dziś udostępnić.

Już dzisiaj cieszymy się na nasze kolejne spotkanie w ramach jubileuszowej XXXV Ogólnopolskiej Konferencji Naukowej im. Profesora Władysława Bukietyńskiego, która tym razem będzie organizowana przez naszych przyjaciół z Katedry Badań Operacyjnych Uniwersytetu Ekonomicznego w Poznaniu pod kierownictwem prof. dr. hab. Krzysztofa Piaseckiego.

Marek Nowiński

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ON TWO APPLICATIONS OF THE OMEGA RATIO: MAXΩMIN AND OMEGA(H+B)

O DWÓCH ZASTOSOWANIACH WSKAŹNIKA OMEGA: MAXΩMIN I OMEGA(H+B)

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Summary: The Omega ratio (Ω-ratio) was proposed by Shadwick and Keating in 2002 as a performance measure applied to rankings of assets, portfolios or funds. The original ratio was developed for decision making under risk (DMR), or decision making under uncertainty with known probabilities (DMUP), i.e. for situations where the probability distribution of particular scenarios is known. The literature reveals that a considerable number of extensions of the Ω-ratio have been suggested recently. Some of them are devoted to decision making with partial information (DMPI), which is characterized by probability distributions known incompletely, and to decision making under complete uncertainty (uncertainty with unknown probabilities) – DMCU. In this contribution, we refer to the maxΩmin decision rule worked out by E. Michalska and to the Omega(H+B) ratio developed by H. Gaspars-Wieloch. Both procedures use two criteria in order to select the optimal decision: the quotient and the difference between weighted profits and losses calculated on the basis of a reference point. The necessity of applying a double criterion is justified especially when weighted profits or losses related to some investments are equal to zero. Nevertheless, in this article, the authors recommend to use an additional, third criterion since in some specific decision situations the first two criteria may turn out to be insufficient. The third criterion enables one to better adjust the final solution to the decision maker's nature.

Keywords: modifications of the Omega ratio, decision making with partial information, decision making under complete uncertainty, decision maker's preferences, reference point.

Streszczenie: Wskaźnik Omega, opracowany przez Shadwicka i Keatinga w roku 2002, znajduje zastosowanie przy ocenie decyzji inwestycyjnych podejmowanych w sytuacji, w których znany jest rozkład prawdopodobieństwa wystąpienia poszczególnych scenariuszy (PDR – podejmowanie decyzji w warunkach ryzyka, PDNP – podejmowanie decyzji w warunkach niepewności ze znanymi prawdopodobieństwami). Miara ta doczekała się wielu modyfikacji, a w ostatnich latach pojawiły się w literaturze prace prezentujące możliwe sposoby konstrukcji wskaźnika Omega w przypadku decyzji podejmowanych przy niepełnej wiedzy o rozkładzie prawdopodobieństwa (podejmowanie decyzji w warunkach niepełnej informacji − PDNI) lub przy braku jakiekolwiek wiedzy o szansie wystąpienia poszczególnych stanów natury (PDN – podejmowanie decyzji w warunkach niepewności, PDCN – podejmowanie decyzji w warunkach całkowitej niepewności, tj. niepewności z nieznanymi prawdopodobieństwami). W artykule przypomniana zostanie idea reguły maxΩmin (dla PDNI) autorstwa E. Michalskiej i reguły Omega(H+B) opracowanej przez H. Gaspars-Wieloch (dla PDCN). Obie procedury wykorzystują dwa kryteria w celu wyłonienia optymalnej decyzji, tj. iloraz oraz różnicę ważonych zysków i strat wyznaczanych na podstawie punktu referencyjnego. Konieczność stosowania podwójnego kryterium jest uzasadniona zwłaszcza wówczas, gdy ważone zyski bądź straty związane z niektórymi inwestycjami są zerowe. Autorki zauważają jednak, iż w niektórych sytuacjach decyzyjnych opieranie się na wspomnianych dwóch kryteriach może się okazać niewystarczające. W artykule zaproponowano trzecie kryterium, dzięki któremu możliwe będzie zawężenie ostatecznego zbioru optymalnych strategii. Zaletą wprowadzenia trzeciego kryterium jest możliwość większego różnicowania rekomendowanych decyzji w zależności od natury decydenta.

Słowa kluczowe: modyfikacje wskaźnika Omega, podejmowanie decyzji w warunkach niepełnej informacji, podejmowanie decyzji w warunkach całkowitej niepewności, preferencje decydenta, punkt referencyjny.

1. Introduction

The Omega ratio (Ω-ratio) was created by Shadwick and Keating [Shadwick, Keating 2002a; 2002b]. It is a performance measure of an investment asset, portfolio or strategy. It involves partitioning returns into gain and loss above and below a given threshold (point of reference). The Ω-ratio constitutes the ratio of expected gains to expected losses. The original Omega was designed for decision making under risk (DMR), that is, for decision problems where the likelihood of particular scenarios (events, states of nature) is known [Sikora (ed.) 2008; Trzaskalik 2008]. There are many extensions of the Ω-ratio [Bargman 2012; Gaspars-Wieloch 2015; Kaplan, Knowles 2004; Kapsos et al. 2014; Kazemi 2004; Michalska 2015]. Some of them [Kapsos et al. 2014; Michalska 2015] are designed for decision making with partial (incomplete, imprecise) information (DMPI), which is characterized by probability distributions known incompletely [Cannon, Kmietowicz 1974; Kofler, Menges 1976]. The measure has been also recently adjusted to decision making under uncertainty with unknown probabilities (DMCU – decision making under complete uncertainty) by [Gaspars-Wieloch 2015]. Within the framework of DMCU the decision maker (DM) has to choose the appropriate alternative (decision, project, strategy) on the basis of some scenarios whose probabilities are not known [Chronopoulos et al. 2011; Knight 1921; Trzaskalik 2008; von Neumann, Morgenstern 1944]. The lack of knowledge concerning probabilities is consistent with the L. von Mises approach. He argues that for an individual event the likelihood cannot be expressed in numbers [von Mises 1962]. In this paper we analyze two procedures referring to the Ω ratio:

the maxΩmin rule worked out by E. Michalska and the Omega(H+B) ratio developed by H. Gaspars-Wieloch. These approaches are designed for different purposes but they have many similar features. They are both based on two criteria: the quotient and the difference between weighted profits and losses. We will demonstrate that in some specific situations these two criteria may prove insufficient and that a third criterion might be necessary. The paper is organized as follows. Section 2 briefly describes the Ω-ratio and its extensions. Section 3 deals with the main features of DMPI and DMCU. Section 4 concerns the original versions of the maxΩmin and Omega(H+B) ratios. Section 5 presents a multiple solutions case. In Section 6 the authors suggest the application of an additional criterion. Section 7 contains the analysis of two case studies. Conclusions are gathered in Section 8.

2. The Omega ratio

The Omega ratio serves to evaluate the performance of an investment within the downside risk framework. It is used to generate rankings of portfolios, funds and assets. It can be applied to portfolio optimization models [Bargman 2012, Kapsos et al. 2014; Mausser et al. 2006] and robust optimization models [Kapsos et al. 2014]. The Omega ratio is computed according to Equation (1):

$$\Omega(r) = \frac{\int_r^b (1 - F(x))dx}{\int_a^r F(x)dx} \tag{1}$$

where $[a,b]$ is the interval of returns, $F(x)$ is the cumulative distribution function and r denotes the threshold (point of reference) defining the gain versus the loss. The reader can find other formulas for the original Ω ratio in [Bargman 2012; Kaplan, Knowles 2004; Michalska 2015]. The ratio should be as big as possible. Omega takes the value 1 when r is the mean outcome. The DM may use as a threshold the accepted wealth level, risk-free rate of interest, stock index rate of return etc. The impact of the reference point level on the final decision is discussed in [Vilkancas 2014]. The measure is relatively simple and does not rely on any assumptions about the distribution of the empirical returns or the shape of the utility function. In contrast to the Sharpe ratio [Sharpe 1966; 1994], where only the first two moments have an influence on the risk measure, the Ω ratio enables taking into account all moments of the distribution. Omega was developed "to overcome the inadequacy of many traditional performance measures applied to investments that do not have normally distributed return distributions" [Shadwick, Keating 2002b]. When calculating Omega, no assumptions about risk preferences or utility are necessary though any may be accommodated.

The standard formulation of the Ω ratio and its first extensions (e.g. Sharpe-Omega [Kazemi et al. 2004], Kappa [Kaplan, Knowles 2004], Omega-H [Bargman

2012]), require perfect information for the probability distribution of the asset returns, which is the case of DMR, but in further research the problem arising from the probability distribution only partially known (DMPI) or completely unknown (DMCU) was also investigated (maxΩmin optimization rule [Michalska 2015], robust variant of the conventional Omega ratio, i.e. the worst-case Ω ratio [Kapsos et al. 2014], and Omega(H+B) ratio [Gaspars-Wieloch 2015]).

In this paper we only refer to the discrete version of the Ω ratio. Omega functions for continuous distributions are investigated e.g. in [Michalska, Dudzińska-Baryła 2015, Michalska, Kopańska-Bródka 2015].

3. Decision making with partial information and under uncertainty with unknown probabilities

It is worth emphasizing that there is no unanimity in defining uncertainty. According to the first approach the DM may choose the appropriate decision under certainty (DMC – each parameter of the problem is deterministic), under risk (DMR), with partial information (DMPI), under complete uncertainty (DMCU) or under total ignorance (DMTI). In the case of DMR, DMPI and DMCU, possible scenarios are predicted by experts or the DM. DMCU occurs when the probability of those events is not known or when the DM does not want to make use of the estimated probabilities. If the likelihood of particular scenarios is known and significant for the DM, then we deal with DMR [Haimann et al. 1985, Knight 1921, Kopańska-Bródka 1998, Sikora 2008, Trzaskalik 2008]. DMPI is characterized by partially known probabilities [Kmietowicz, Pearman 1984, Kofler, Zwiefel 1993], which means that the DM knows only a) the order of scenarios or b) the intervals with possible probabilities for each scenario. DMTI concerns problems for which the DM is not able to define possible events. Uncertainty and risk were formally integrated in economic theory by [von Neumann, Morgenstern 1944].

Supporters of the second approach state that uncertainty involves all situations with non-deterministic parameters, while risk means the possibility that some bad circumstances happen (potential of losing something) [Dominiak 2009; Dubois, Prade 2012; Fishburn 1984; Ogryczak, Sliwinski 2009; Oxford English Dict.].

In both approaches scenarios may be related to different fields, see e.g. weather scenarios, market scenarios, economic scenarios etc.

In this paper we treat uncertainty according to the first approach. We focus on DMCU and DMPI. DMCU can be presented by means of a payoff matrix, where m (the number of rows) denotes the number of mutually exclusive scenarios (S_1, …S_i, … S_m), n (the number of columns) stands for the number of decisions (D_1, …, D_j, …, D_n) and a_{ij} is the profit connected with S_i and D_j. In the case of DMPI there is an additional column concerning the likelihood of particular states of nature.

DMPI starts with the analysis of the probability distribution of the considered events. All possible probability distributions $\mathbf{p} = (p_1, p_2, ..., p_m)$ make up a so-called simplex of distributions. Considering two states of nature S_1 and S_2, without any additional information about the probabilities of their occurrence, the probability distributions $\mathbf{p} = (p_1, p_2)$ correspond to the points of a segment in a two-dimensional space. If three scenarios S_1, S_2, S_3 are considered, distributions $\mathbf{p} = (p_1, p_2, p_3)$ make up the set of points of a triangle in a three-dimensional space and for m scenarios we obtain the set of all points of a proper simplex in a m-dimensional space [Kofler 1968]. All possible probability distributions make up the set being a convex polyhedron. Therefore, if the criterion for choosing the optimal decision is based on the linear function, then the extreme values are attained in the vertices of that set. When partial information about probabilities $p_1, ..., p_m$ is expressed in the form of linear relationships e.g. $p_1 \geq ... \geq p_m$, $p_1 + p_2 + ... + p_m = 1$ and $p_i \geq 0$ for $i = 1, ..., m$, then distributions $\mathbf{p}^{(1)}, ..., \mathbf{p}^{(v)}$ are vertices of the simplex of distributions that corresponds to solutions of the system of linear equations and (or) inequalities.

The literature offers numerous decision rules for DMPI [Guo 2013; 2014; Kmietowicz, Pearman 1984; Kofler, Zweifel 1993; Michalska, Pośpiech 2010; 2011, Michalska 2012; 2015; Weber 1987] and for DMCU [Gaspars 2007; Gaspars-Wieloch 2013; 2014a; 2014b; 2014c; 2014d; 2014e; 2015a; 2015b; 2015c; 2015d; 2015e; 2016; Hayashi 2008; Hurwicz 1952; Ioan, Ioan 2011; Piasecki 1990; Savage 1961; Wald 1950], but here we only investigate the use of the Ω ratio in these two decision cases.

4. MaxΩmin and Omega(H+B) as two-criteria decision rules

As was mentioned in the introduction, the contribution concerns maxΩmin and Omega(H+B) which are both two-criteria decision rules based on the Ω ratio. The maxΩmin approach is designed for DMPI and it may be only applied by pessimists. The Omega(H+B) procedure was invented for DMCU, but it may be used by any DM (pessimist, optimist, moderate). Both measures, like the original Ω ratio does, enable one to define the reference point. Such a possibility also occurs e.g. [Tversky, Kahneman 1992]. A detailed description of maxΩmin and Ω(H+B) is presented in [Michalska 2015, Gaspars-Wieloch 2015], respectively. In this section we just recall the main features of both methods.

In the case of maxΩmin, one ought to find, for each decision, such a probability distribution (vertex of the simplex of distributions) that minimizes the Ω-ratio, and then the decision with the highest value of the minimal Ω-ratio is selected. A similar approach is applied for instance in the Wald decision rule [Wald 1950]. The use of maxΩmin leads to finding the optimal solution of the following optimization model:

$$\max_j \min_{\mathbf{p}} \Omega_j(r) = \frac{E(\mathbf{g}_j)}{E(\mathbf{l}_j)} \tag{2}$$

$$\mathbf{A}\mathbf{p}^T \leq \mathbf{b} \tag{3}$$

$$\mathbf{Jp}^T = 1 \quad (4)$$

$$\mathbf{p} \geq \mathbf{\theta} \quad (5)$$

where $\mathbf{p} = (p_1, \ldots, p_m)$ is the vector of probabilities that particular scenarios will occur, r denotes the reference point, $\Omega_j(r)$ signifies the Omega ratio (expected gains $E(\mathbf{g}_j)$ divided by expected losses $E(\mathbf{l}_j)$) for D_j and distribution **p**. Symbol **A** stands for the coefficient matrix of a system of linear constraints concerning probabilities $p_1, \ldots, p_m$. Symbol **b** denotes the vector of constant terms for linear constraints, **J** and $\mathbf{\theta}$ are vectors of m elements equal to 1 and 0, respectively.

The use of the Ω(H+B) rule is a little more complex, since, apart from the reference point (r), it takes into account DM's preferences measured by the coefficients of pessimism (α) and optimism (β): $\alpha, \beta \in [0,1]$, $\alpha + \beta = 1$. The Ω(H+B) approach combines the original Ω-ratio with the Bayes and Hurwicz rules (the Bayes rule takes into consideration all outcomes, and the Hurwicz rule is based on the coefficients of pessimism and optimism). Parameters α and β concern DM's predictions and r describes his/her aspirations. After computing all the relative outcomes (on the basis of the reference point), the numerator and the denominator of the Ω(H+B) ratio are calculated. The way it is done depends on the DM's nature. For pessimists, α is assigned to the biggest loss and β is multiplied by all the remaining relative outcomes (all gains and almost all losses). For optimists, β is assigned to the biggest profit and α is multiplied by all the remaining relative outcomes (all losses and almost all gains). The choice of such weights is justified in [Gaspars-Wieloch 2014; 2015; 2016]. The rule recommends the alternative with the highest value of the Ω(H+B) ratio.

It is worth underlining that the numerator: N (or the denominator: DN) of the Ω fraction may be equal to zero, for instance when all the outcomes connected with a decision are lower (or higher) than the reference point. This leads to a situation where the Omega and Omega(H+B) ratios of particular alternatives are not comparable. That is why some rules have been proposed by [Michalska 2015] to come up with this impediment. These rules have been also implemented in the Ω(H+B) approach by [Gaspars-Wieloch 2015]. They consist in dividing all decisions into three groups: (A) alternatives with gains and losses (both N and DN of the Ω-ratio are positive), (B) alternatives with no losses, (C) alternatives with no gains. The division is followed by using the Omega fraction only for decisions containing gains and losses. For the remaining alternatives merely the positive N or DN is taken into consideration. After appointing the best decision in each group separately (in A – with the highest quotient, in B – with the highest N, in C – with the lowest DN), an additional criterion (being the difference between weighted gains and losses) is introduced to compare the "winners" from particular groups and select the final decision (with the biggest difference). Therefore both methods are two-criteria decision rules since they are based on the quotient and difference criterion.

5. The multiple solution case

Generally, the rules suggested in [Michalska 2015], enable avoiding the occurrence of more than one optimal alternative. Nevertheless, there are decision problems for which both procedures (maxΩmin and Ω(H+B)) suggest a multi-element set of optimal decisions and, which is astonishing and alarming, those variants, despite their "optimality" and "equivalence", differ significantly.

Let us analyze Example 1 presented in Table 1. The DM considers six decisions. The sums of all weighted gains and losses are given in columns 2 and 3. Due to some zero numerators and denominators, we assign D1-D2 to group C, D3-D4 to group B, and D5-D6 to group A (column 4). There are three winners according to the first criterion (D1 from group C with the lowest DN, D3 from group B with the highest N and D6 from group A with the highest quotient), see column 5. Hence, the second criterion is going to be applied to three variants. Alternatives D3 and D6 have the highest and the same difference between gains and losses. According to the maxΩmin and Ω(H+B) approaches there are two optimal decisions (column 6). The occurrence of the two final solutions seems to be absolutely normal. However, the relation between the gains and losses for these two variants turns out to be extremely diverse. Their distribution of the relative outcomes are totally different. If the DM is cautious, he (or she) will be certainly more willing to perform D3 (with gains equal to 1 and losses equal to 0) than D6 (gains equal 100 and losses equal 99!). D3 is much safer. Thus, those variants are not equivalent. Currently, neither the maxΩmin nor Omega(H+B) approaches are ready to cope with such a decision problem. A similar example is demonstrated in Table 2 (Example 2). This time, the winners come only from groups A and C, and again two decisions are treated as optimal: D1 and D6. Nevertheless, D6 should not be recommended for a pessimist as in the case of D1, the losses are much lower. Hence, D1 better corresponds to the pessimist's nature. We clearly notice that in some specific situations the use of two criteria (quotient and difference) is not sufficient to find a suitable optimal solution. That is why we propose an extended version of maxΩmin and Omega(H+B) in Section 6.

Table 1. Example 1

Decision	Gain	Loss	Group	Value of the I criterion	Value of the II criterion
D1	0	1	(C)	1 (winner)	0-1 = –1
D2	0	2	(C)	2	
D3	1	0	(B)	1 (winner)	1-0 = 1 (optimal)
D4	0.5	0	(B)	0.5	
D5	1	2	(A)	½ = 0.5	
D6	100	99	(A)	100/99 = 1,0101 (Winner)	100-99 = 1 (optimal)

Source: created by the authors.

Table 2. Example 2

Decision	Gain	Loss	Group	Value of the I criterion	Value of the II criterion
D1	0	1	(C)	1 (winner)	0-1 = –1 (optimal)
D2	0	2	(C)	2	
D3	1	2	(A)	½ = 0.5	
D4	99	100	(A)	99/100 = 0.990000	
D5	100	101	(A)	100/101 = 0.9901	
D6	101	102	(A)	101/102 = 0.9902 (winner)	101-102 = –1 (optimal)

Source: created by the authors.

6. MaxΩmin and Omega(H+B) as three-criteria decision rules

The extended three-criteria maxΩmin approach consists of the following steps:

1. Define a reference point (r).
2. Transform initial outcomes (a_{ij}) into relative outcomes (a^r_{ij}) with respect to r:

$$a^r_{ij} = a_{ij} - r \quad i = 1,\dots,m;\ j = 1,\dots,n \ . \tag{6}$$

3. For each decision D_j ($j = 1,\dots,n$) determine vectors $\mathbf{g}_j$ and $\mathbf{l}_j$:

$$\mathbf{g}_j = [g_{ij}] \quad i = 1,\dots,m;\ j = 1,\dots,n \tag{7}$$

$$\mathbf{l}_j = [l_{ij}] \quad i = 1,\dots,m;\ j = 1,\dots,n \tag{8}$$

where $g_{ij} = \begin{cases} a^r_{ij} & \text{for} \quad a_{ij} > r \\ 0 & \text{for} \quad a_{ij} \le r \end{cases}$ and $l_{ij} = \begin{cases} |a^r_{ij}| & \text{for} \quad a_{ij} < r \\ 0 & \text{for} \quad a_{ij} \ge r \end{cases}$.

4. Divide all decisions into three groups: (A) – decisions which generate profits and losses, (B) – decisions with no losses, (C) – decisions with no gains.

5. In each group indicate the best alternative among the worst by using the decision criterion (model) proper for each group as follows:

(A)

$$\max_j \min_p \frac{E(\mathbf{g}_j)}{E(\mathbf{l}_j)}$$

$$\mathbf{Ap}^{\mathrm{T}} \le \mathbf{b}$$

$$\mathbf{Jp}^{\mathrm{T}} = 1$$

$$\mathbf{p} \ge \mathbf{\theta}$$

(B)

$$\max_j \min_p E(\mathbf{g}_j)$$

$$\mathbf{Ap}^{\mathrm{T}} \le \mathbf{b}$$

$$\mathbf{Jp}^{\mathrm{T}} = 1$$

$$\mathbf{p} \ge \mathbf{\theta}$$

(C)

$$\min_j \max_p E(\mathbf{l}_j)$$

$$\mathbf{Ap}^{\mathrm{T}} \le \mathbf{b}$$

$$\mathbf{Jp}^{\mathrm{T}} = 1$$

$$\mathbf{p} \ge \mathbf{\theta}$$

Determine D_{I} – the set of the best variants selected in all groups. If D_{I} is a singleton, stop (the best solution $D_{j*} \in D_I$ is found). Otherwise, go to step 6.

6. Choose the best alternative $D_{j^{**}}$ from D_{I} using the following criterion:

$$D_{j^{**}} = \arg\max_{D_j \in D_I} \{E(\mathbf{g}_j) - E(\mathbf{l}_j)\} \ . \tag{9}$$

If more than one alternative satisfies condition (9), create set D_{II} containing these alternatives and go to step 7. Otherwise, stop – solution $D_{j^{**}}$ is optimal.

7. Choose the best alternative from D_{II} according to the following criterion:

$$D_{j^{***}} = \arg\min_{D_j \in D_I} \{E(\mathbf{l}_j)\} \ . \tag{10}$$

If more than one alternative is chosen according to criterion (10) (set D_{III} includes more than one element), then all of them are optimal.

The extended three-criteria Omega(H+B) approach is based on the following steps:

1. Determine α, i.e. the DM's coefficient of pessimism. If $\alpha \in [0,0.5)$, then $\alpha = \alpha_o, \beta = \beta_o$ (α_o and β_o are optimist's coefficients). If $\alpha \in (0.5,1]$, then $\alpha = \alpha_p, \beta = \beta_p$ (α_p and β_p are pessimist's coefficients).

2. Define the point of reference (r).

3. Transform initial outcomes (a_{ij}) into relative outcomes (a^r_{ij}) using Equation 6.

4. Define a non-increasing sequence of gains $Sq(G)_j = (g_{1j},...,g_{uj},...,g_{xj})$ for each decision, where $g_{u,j} \geq g_{u+1,j}$ ($u = 1, \ldots, x-1$), and a non-decreasing sequence of losses $Sq(L)_j = (l_{1j},...,l_{wj},...,l_{zj})$, $l_{w,j} \leq l_{w+1,j}$ ($w = 1, \ldots, z-1$):

$$a^r_{ij} > 0 \Rightarrow a^r_{ij} = g_{uj} \in Sq(G)_j \quad j = 1,...,n; \ i = 1,...,m \tag{11}$$

$$a^r_{ij} < 0 \Rightarrow \left|a^r_{ij}\right| = l_{wj} \in Sq(L)_j \quad j = 1,...,n; \ i = 1,...,m \ . \tag{12}$$

Notice that $0 \leq x + z \leq m$ (x – number of gains, z – number of losses).

5. For each decision compute numerator N_j and denominator Dn_j:

a. If $1 < x < m$, $1 \leq z < m$ and $\alpha \in [0,0.5)$:

$$N_j = \beta_o \cdot g_{1j} + \alpha_o \cdot \sum_{u=2}^{x} g_{uj} \ , \ Dn_j = \alpha_o \cdot \sum_{w=1}^{z} l_{wj} \quad j = 1,...,n \ . \tag{13}$$

b. If $x = 1$, $1 \leq z < m$ and $\alpha \in [0,0.5)$:

$$N_j = \beta_o \cdot g_{1j} \ , \ Dn_j = \alpha_o \cdot \sum_{w=1}^{z} l_{wj} \quad j = 1,...,n \ . \tag{14}$$

c. If $x = 0$, $1 \leq z \leq m$ and $\alpha \in [0,0.5)$:

$$N_j = 0 ,\ Dn_j = \beta_o \cdot l_{1j} + \alpha_o \cdot \sum_{w=2}^{z} l_{wj} \quad j = 1,...,n . \tag{15}$$

d. If $1 \le x \le m$, $z = 0$ and $\alpha \in [0,0.5)$:

$$N_j = \beta_o \cdot g_{1j} + \alpha_o \cdot \sum_{u=2}^{x} g_{uj} ,\ Dn_j = 0 \quad j = 1,...,n . \tag{16}$$

e. If $1 \le x < m$, $1 < z < m$ and $\alpha \in (0.5,1]$:

$$N_j = \beta_p \cdot \sum_{u=1}^{x} g_{uj} ,\ Dn_j = \beta_p \cdot \sum_{w=1}^{z-1} l_{wj} + \alpha_p \cdot l_{zj} \quad j = 1,...,n . \tag{17}$$

f. If $1 \le x < m$, $z = 1$ and $\alpha \in (0.5,1]$:

$$N_j = \beta_p \cdot \sum_{u=1}^{x} g_{uj} ,\ Dn_j = \alpha_p \cdot l_{1j} \quad j = 1,...,n . \tag{18}$$

g. If $x = 0$, $1 \le z \le m$ and $\alpha \in (0.5,1]$:

$$N_j = 0 ,\ Dn_j = \beta_p \cdot \sum_{w=1}^{z-1} l_{wj} + \alpha_p \cdot l_{zj} \quad j = 1,...,n . \tag{19}$$

h. If $1 \le x \le m$, $z = 0$ and $\alpha \in (0.5,1]$:

$$N_j = \beta_p \cdot \sum_{u=1}^{x-1} g_{uj} + \alpha_p \cdot g_{xj} ,\ Dn_j = 0 \quad j = 1,...,n . \tag{20}$$

i. If $\alpha = 0.5$, one can use any formulas (Equations (13)-(20)) depending on *x* and *z*. Regardless of the applied formula, N_j is equal to the sum of gains and Dn_j is the sum of losses.

Divide all decisions D_j into three groups: (A) decisions with a positive numerator (N) and denominator (DN), (B) decisions with a positive N and a 0-DN, (C) decisions with a 0-N and a positive DN:

$$(N_j > 0) \wedge (Dn_j > 0) \Rightarrow D_j \in A \tag{21}$$

$$(N_j > 0) \wedge (Dn_j = 0) \Rightarrow D_j \in B \tag{22}$$

$$(N_j = 0) \wedge (Dn_j > 0) \Rightarrow D_j \in C . \tag{23}$$

6. Find the best variant in each group (choose decisions fulfilling formulas (24)-(26)) and determine D_{I} – the set of the best variants from each group. If D_{I} is a singleton, stop (the best solution is found). Otherwise, go to step 7.

$$\Omega(hb)_{j*}^{A} = \max_{D_j \in A}\left\{\frac{N_j}{Dn_j}\right\} \tag{24}$$

$$\Omega(hb)_{j*}^{B} = \max_{D_j \in B}\left\{N_j\right\} \tag{25}$$

$$\Omega(hb)_{j*}^{C} = \min_{D_j \in C}\left\{Dn_j\right\} \tag{26}$$

7. Find the optimal decision by means of an additional criterion (Equation 27) used only for decisions selected in step 6 (i.e. belonging to D_{I}):

$$D_{j**} = \arg\max_{D_j \in D_I}\left\{N_j - Dn_j\right\} . \tag{27}$$

If more than one alternative fulfills condition (27), go to step 8 (set D_{II} containing solutions selected by means of two criteria is not a singleton). Otherwise, stop – the solution is optimal.

8. a. Choose, at one's discretion, the most appropriate alternative belonging to D_{II} (analyze both weighted gains and losses, and all relative outcomes connected with potential final variants). If the choice is not easy and obvious, go to step 8b (for pessimists and moderate DMs) or 8c (for optimists).

b. If $\alpha \in [0.5,1]$, find the optimal alternative on the basis of Equation (28):

$$D_{j***} = \arg\min_{D_j \in D_{II}}\left\{Dn_j\right\} . \tag{28}$$

c. If $\alpha \in [0,0.5)$, find the optimal alternative on the basis of Equation (29):

$$D_{j***} = \arg\max_{D_j \in D_{II}}\left\{N_j\right\} . \tag{29}$$

If more than one decision satisfies (29) or (30), set D_{III} containing solutions selected by means of three criteria is not a singleton. However again, it is recommended to choose the final decision at one's discretion.

The third criterion used for Omega(H+B) is slightly different from the measure applied as a third criterion in maxΩmin because the Ω(H+B) approach is designed for all kinds of DMs and the maxΩmin rule is only addressed to prudent DMs. In the case of Ω(H+B), due to diverse DM's preferences, there is also a possibility to select the final decision after the use of the first two criteria, at one's discretion.

7. Case studies

The three-criteria maxΩmin rule will be demonstrated in Example 3. The DM considers three decisions. Their outcomes depend on scenarios: S1, S2, S3. He/she has chosen the threshold level $r = 4$. Let us assume that partial information about probabilities p_1, p_2, p_3 of events is as follows: $p_1 \geq p_2 \geq p_3 \geq 0.25$; $p_1 + p_2 + p_3 = 1$; p_1, p_2,

$p_3 \geq 0$. The vertices of such a feasible set of distributions are $\mathbf{p}^{(1)} = (1/3;1/3;1/3)$, $\mathbf{p}^{(2)} = (2/4;1/4;1/4)$ and $\mathbf{p}^{(3)} = (3/8;3/8;1/4)$ (an example of developing partial information about scenario probabilities is presented in [Michalska 2015]). Outcomes and relative outcomes are shown in Table 3. Table 4 presents gains and losses for each decision. D1 and D2 belong to group A (they generate both gains and losses). D3 does not generate losses – it belongs to group B (Table 5). Using a suitable model for a given group of decisions, we determine the best alternative in each group: D1 in A and D3 in B: $D_{\mathrm{I}} = \{D1,D3\}$. Criterion (9) helps in deciding which one is better. Both differences between expected gains and losses are equal: $D_{\mathrm{II}} = \{D1,D3\}$. According to the next criterion (10), set D_{III} includes only one element D3 which is the best alternative.

Table 3. Payoff matrix. Example 3

Values	Outcomes			Prob{S_j}	Relative outcomes		
Scenarios \ Decisions	D1	D2	D3		D1	D2	D3
S1	3	2	6	P_1	–1	–2	2
S2	5	9	5	P_2	1	5	1
S3	9	3	4	P_3	5	–1	0

Source: created by the authors.

Table 4. Gains and losses. Example 3

Values	Gains $\mathbf{g}_j$			losses $\mathbf{l}_j$		
Scenarios \ Decisions	D1	D2	D3	D1	D2	D3
S1	0	0	2	1	2	0
S2	1	5	1	0	0	0
S3	5	0	0	0	1	0

Source: created by the authors.

Table 5. Three-criteria Omega ratios. Example 3

Decisions	D1	D2	D3
Group	(A)	(A)	(B)
$E[\mathbf{g}_j]$	1.5	1.25	1.0
$E[\mathbf{l}_j]$	0.5	1.25	0.0
Optimal **p**	**p** = (2/4;1/4;1/4)	**p** = (2/4;1/4;1/4)	**P** = (1/3;1/3;1/3)
Criterion I	3.0	1.0	1.0
Criterion II	**1.0**	–	**1.0**
Criterion III	0.5	–	0.0

Source: created by the authors.

Now, let us illustrate the application of the three-criteria Ω(H+B) ratio. In Example 4 the DM can select one out of four decisions. One knows that one out of five scenarios will occur in the future. The possible outcomes and relative outcomes (for $r = 10$) dependent on the chosen alternative and on the true event are given in Table 6. Assume that the DM's coefficient of pessimism amounts to $\alpha = 0.7$. Hence, the DM is a moderate pessimist. The first two rows of Table 7 contain the values of the numerators and denominators. The denominators for D3 and D4 are equal to zero because those decisions have no losses. In the third row the alternatives have been assigned to an appropriate group: A ={D1,D2}, B ={D3,D4}, C ={∅}. The fourth and fifth row present the Ω(H+B) ratio. D1 is the best decision within group A and D4 is the best decision among the alternatives belonging to B: D_{I} ={D1,D4}. Now let us compute the second measure for the two winners (sixth row) – values are equal: D_{II} ={D1,D4}. Thus, the DM can select one of those variants by means of the analysis of weighted gains, weighted losses and all the relative outcomes, or one may apply the third criterion which, for pessimists, consists in minimizing weighted losses (seventh row). The DM ought to perform decision D4: D_{III} ={D4}. Note that for a moderate optimist (e.g. $\alpha = 0.3$), particular sets contain the following decisions: D_{I} ={D2,D4}, D_{II} ={D2}. This time the use of the third criterion is not necessary.

Table 6. Payoff and relative outcome matrix. Example 4

Values	Outcomes				Relative outcomes ($r = 10$)			
Scenarios \ Decisions	D1	D2	D3	D4	D1	D2	D3	D4
S1	52	60	15	18	42	50	5	8
S2	40	30	13	13	30	20	3	3
S3	10	9	11	11	0	–1	1	1
S4	–2	8	10	10	–12	–2	0	0
S5	–10	–20	10	10	–20	–30	0	0

Source: created by the authors.

Table 7. Three-criteria Omega(H+B) ratios ($\alpha = 0.4$). Example 4

Decisions	D1	D2	D3	D4
N_j	21.6	21.0	3.1	4.0
Dn_j	17.6	21.9	0.0	0.0
Group	A	A	B	B
$\Omega(hb)^A_j = N_j/Dn_j$ (I criterion)	1.227	0.959	–	–
$\Omega(hb)^B_j = N_j$ (I criterion)	–	–	3.1	4.0
N_j-Dn_j (II criterion)	**4.0**	–	–	**4.0**
$Dn_j \rightarrow \min$ (III criterion for pessimists)	17.6	–	–	0

Source: created by the authors.

8. Conclusions

The maxΩmin and Ω(H+B) approaches are based on the original version of the Omega ratio. The maxΩmin decision rule is designed for decision making with partial information and it may support cautious DMs. The Ω(H+B) procedure may be applied by any DM on condition that the decision is made under uncertainty with unknown probabilities. Both methods are two-criteria decisions rules. They are described in detail in [Michalska 2015] and [Gaspars-Wieloch 2015], respectively. Nevertheless, recent research made by the authors has revealed that in some very specific decision problems (multiple solutions case), the use of two criteria (quotient and difference between gains and losses) is not sufficient. Therefore in this contribution, a third measure has been proposed separately for each considered procedure. This criterion focuses on gains or losses, depending on the nature of the DM. As a matter of fact, there are several other ways to appoint the final alternative in the multiple solutions case, e.g. one may refer to the standard deviation [Ioan, Ioan 2011; Gaspars-Wieloch 2015c; 2016].

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