DE GRUYTER OPEN
Studia Geotechnica et Mechanica, Vol. 38, No. 1, 2016
DOI: 10.1515/sgem-2016-0002

NUMERICAL ANALYSIS OF ROAD PAVEMENT THERMAL DEFORMABILITY, BASED ON BIOT VISCOELASTIC MODEL OF POROUS MEDIUM

MONIKA BARTLEWSKA-URBAN, MAREK ZOMBROŃ

Wrocław University of Science and Technology,
Faculty of Geoengineering, Mining and Geology, ul. Na Grobli 15, 50-421 Wrocław, Poland,
e-mail: monika.bartlewska@pwr.wroc.pl

TOMASZ STRZELECKI

Wrocław University of Science and Technology

Abstract: The following study presents numerical calculations for establishing the impact of temperature changes on the process of distortion of bi-phase medium represented using Biot consolidation equations with Kelvin–Voigt rheological skeleton presented, on the example of thermo-consolidation of a pavement of expressway S17. We analyzed the behavior of the expressway under the action of its own weight, dynamic load caused by traffic and temperature gradient. This paper presents the application of the Biot consolidation model with the Kelvin–Voigt skeleton rheological characteristics and the influence of temperature on the deformation process is taken into account. A three-dimensional model of the medium was created describing the thermal consolidation of a porous medium. The 3D geometrical model of the area under investigation was based on data obtained from the land surveying and soil investigation of a 200 m long section of the expressway and its shoulders.

Key words: *Biot theory, thermal consolidation, rheological body of Kelvin–Voigt, road pavement*

1. INTRODUCTION

The notion of safe, durable and good road covers a wide range of different problems. From the users' point of view, the road can be regarded as a transport route while from the technical angle, it is a structural system constituting a construction. The road structure comprises a structural pavement system consisting of many courses, and a foundation (an earthen structure + a pavement subgrade). The safety and durability of building or civil engineering structures to a large extent depend on the limit loads or the service life of their components. As regards the two main components distinguished above, the principal factor is the service quality life of the pavement structural system, determining the usability of the road. The symptoms of reduced road usability include ruts and pavement cracks and heaves. According to Gradkowski [1], this aspect of the pavement structure has been insufficiently standardized and there are no standard ways of calculating the multilayer system comprising the road pavement subgrade. It should be noted that the catalogue of pavement structures or the equivalent construction specifications cannot be used as the basis for the design of pavement durability since they do not include the parameters of the pavement and subgrade component materials, stemming from the adopted rheological models and from taking into account the liquid/gaseous phase in the pores of the medium and the dynamic effects of the road loading. Another reason are the deficiencies of the current pavement dimensioning methods and the lack of adequate physical models which would include these parameters [1].

1.1. ANALYTICAL MODELS OF ROAD PAVEMENTS

The pavement of motor roads is a civil engineering structure the function of which is to take over static and dynamic loads originating from vehicles and other external (e.g., thermal) impacts and transfer them to subgrade or another civil engineering structure. It was the appearance of lorries on roads which demonstrated the damaging effect of their wheel load. This gave rise to the first theories about the distribution of stresses in the pavement and in the subgrade. Each road pavement has a layered structure. The layers (courses) can be divided into three groups: those of the pavement proper (the wear and binder courses), those of the base (the road base strengthening courses, the road base top and bottom courses) and those of the subgrade (the treated zone and the natural subgrade courses). Because of its characteristics, the road pavement should function first of all in the range of elastic deformations. Asphalt pavements function also after the exceedance of the elastic limit (i.e., in plastic states), showing viscous properties in both the elastic range and the plastic one. Thus the viscosity of a flexible (asphalt) pavement is one of the principal mechanical parameters of a mathematical pavement model. Each road pavement rests directly on the subgrade or a pre-prepared civil engineering structure. The main components of soils are: soil matrix and pores filled with water or with water and air. For over a century various physical and mathematical models of the subgrade, describing approximately the behaviour of a real soil medium, have been created. Soil is a multiphase medium since it consists of dry soil particles and water or gas. But in soil mechanics one- or two-phase models are usually used. All these models as applied to the pavement and the subgrade are used in the dynamic and static analysis and design of road pavements. There are several thousand works, including many monographs and overview papers on the subject. Also the rheological properties of soils and rocks are the subject of numerous publications and research projects. Rheological models of continuous medium mechanics are usually used to describe the processes involved. As the starting point the models assume a multiphase medium model according to which the solid body contains hydraulically connected pores or microgaps enabling the filtration flow of a liquid and/or a gas. A mathematical creep model of a porous medium defined as a multicomponent two-phase body was for the first time introduced by Biot in 1955 and 1956 [2], [3]. The authors of the present paper propose to represent the thermal consolidation of the road pavement together with the subgrade by means of the Biot model, taking into account the Kelvin–Voigt skeleton rheological properties. The three-dimensional model contains generalized Biot equations for any non-isothermal processes. The displacement of the road pavement over time versus variable temperature was analyzed on the basis of the annual distribution of temperatures prevailing in the area of the modelled road. The temperature distribution was presented using the sine function.

2. LOCATION OF INVESTIGATED OBJECT – SECTION OF EXPRESSWAY S17

This paper presents the next (after the Żelazny Most flotation tailings disposal lake [4]) application of the Biot consolidation model with the Kelvin–Voigt skeleton rheological characteristics and the influence of temperature on the deformation process taken into account. The subject of the analysis was a 200 m long section of the existing 14 km long expressway S17 built by Dragados S.A. The geological data (Fig. 1) were made available by the GGDKiA (General Directorate for National Roads and Motorways) Branch in Lublin.

Fig. 1. Geological map showing location of analyzed section of expressway S17 [7]

The area investigated is located within the administrative boundaries of Niemce, Wólka Lubelska and Lublin in the Lublin District in the Lublin Province. According to the physical-geographical regionalizaton [5], the route of the planned road section runs within the Lublin Upland. As regards its morphology, the area covers the Pliocene high plain. Considering its utilitarian use, the area consists mostly of agricultural land and some wasteland. Residential developments are situated mainly in the vicinity of larger intersections. As already mentioned, a 200 m long section of expressway S17 near the Lubartów interchange was modelled, taking into account the topography and geology of the area (acquired from the project documentation). The effective parameters of the model were developed on the basis of one of the authors own research [6]. The other parameters, including the thermal ones, were taken from the literature.

According to the geological map of the surface deposits, the area of the planned road consists mainly of clays, silts and sandstones, marls and tertiary limestones and quaternary loesses and aggradate muds.

3. EQUATIONS OF THERMAL CONSOLIDATION

The following mathematical equations of thermal consolidation for the Biot body with a rheological skeleton were derived from the laws of Newtonian mechanics for continuous media and from irreversible thermodynamics. The starting point were the input assumptions of the theory of two-phase media consisting of an elastic-viscous skeleton and a compressible viscous fluid filling the pores of the medium. The assumptions are described in detail in [2]. The detailed derivation of the equations is provided in other papers which are in the process of publication. Starting from the equations of the motion of the liquid phase and the solid phase and the equations of the flow of the liquid through the soil matrix, the following constitutive relations for the Biot body with the Kelvin–Voigt rheological skeleton were obtained for any non-isothermal processes

$$\begin{cases} \sigma_{ij} = 2N\varepsilon_{ij} + M\varepsilon\delta_{ij} + 2NT_a\dot{\varepsilon}_{ij} + (AT_b + NT_a)\dot{\varepsilon}\delta_{ij} \\ \qquad + \dfrac{Q}{R}\sigma\delta_{ij} + P_1(T - T_0)\delta_{ij}, \\ \sigma = Q\varepsilon + RQ + d(T - T_0), \end{cases} \tag{1}$$

where N is a modulus of skeleton elasticity in shear, A – a modulus of volume elasticity, Q – a coefficient of the influence of the liquid dilatational strain on the stress in the skeleton or vice versa a coefficient of the influence of the skeleton dilatational strain on the stress in the liquid, R – a modulus of the volume elasticity of the liquid filling the pores of the Biot body. Parameter M is expressed by

$$M = A - \frac{Q^2}{R}. \tag{2}$$

Constant d is expressed by the formula

$$d = -[3Qr^s + r^l R] \tag{3}$$

where r^s and r^l represent, the linear expansion of the skeleton and the volume expansion of the liquid, respectively; P_1 is calculated from the formula

$$P_1 = -\frac{T(3Kr^s + Qr^l)}{\lambda} \tag{4}$$

where λ stands for the thermal permeability of the soil, and T_a and T_b stand for the skeleton parameters expressed by the formulas

$$T_a = \frac{\eta^s}{N} \quad \text{and} \quad T_b = \frac{\lambda^s}{A}$$

where η^s, λ^s – the non-dilatational viscosity and bulk viscosity of the soil matrix, respectively.

The overall system of equations of the linear theory of thermal consolidation for the Biot body with the Kelvin–Voigt rheological skeleton consists of the following five differential equations (equations of skeleton displacements and of the function of stress σ in the liquid, filtration flow equations and heat conduction equations)

$$\begin{cases} N\Psi_k\nabla^2 u_i + \left(A\Psi_L - \dfrac{Q^2}{R} + N\Psi_k\right)\varepsilon,_i + \dfrac{H}{R}\sigma,_i \\ -\rho g\delta_{i3} = -P_1 T,_i \quad \dfrac{kR}{f_o^2\bar{\rho}g}\nabla^2\sigma = T_o[\dot{\sigma} - H\dot{\varepsilon} + P_4\dot{T}], \\ \lambda\nabla^2 T = T_0[P_2\dot{\varepsilon} - P_3\dot{\sigma} + P_5\dot{T}] \end{cases} \tag{5}$$

where $\Psi_k = 1 + T_a\dfrac{\partial}{\partial t}$, $\Psi_L = 1 + T_b\dfrac{\partial}{\partial t}$ are differential operators, k is a coefficient of fluid filtration through a porous medium, g – acceleration due to gravity, and coefficients P_2, P_3, P_4 and P_5 are expressed by the formulas

$$\begin{gathered} P_2 = 3r^s\left(K - \frac{HQ}{R}\right) - Rr^l, \quad P_3 = 3r^s\frac{Q}{R} + r^l, \\ P_4 = RP_3, \quad P_5 = \frac{(3Qr^s + r^l R)^2}{R} + \frac{(\bar{\rho}_s + \bar{\rho}_w)c_v}{T}, \end{gathered} \tag{6}$$

where c_v stands for specific heat at a constant volume. The above system of equations is the starting point for the solution of the problem considered.

4. BUILDING THREE-DIMENSIONAL NUMERICAL MODEL

OF THERMAL CONSOLIDATION OF EXPRESSWAY S17

A three-dimensional model of the medium was created on the basis of the system of equations (5) describing the thermal consolidation of a porous medium. Computations were performed using the finite element method and the Flex PDE v. 6 Professional software [8]. The experimental physical and strength parameters and the thermal parameters taken from the literature were used to generate a 3D numerical consolidation model for the road pavement and the subsoil on the basis of the analytical Biot model with the Kelvin–Voigt rheological skeleton. In order to obtain the effective parameters of the Biot model with the Kelvin–Voigt rheological skeleton calibration was performed using the statistical methods presented in [9]. Since the preliminary analysis of model identifiability had been carried out on the model equations, three parameters were subjected to calibration. The geology and geometry of the terrain were taken from the geological documentation of the planned section of expressway S17.

The whole area subjected to the computer simulation of the thermal consolidation process consists of three geological layers (silty clay, silty clay + residual clay soil and aggregate mud) extending to a depth of about 3 m and two structural pavement courses (the base and the asphalt) making up the road considered. The expressway carriageways are separated by a green central reservation. The simulation covers also the road shoulders. The first step in building a thermal consolidation model most accurately reflecting the real conditions prevailing on the road is a thorough analysis of the data on the geometry of the area under investigation. In the case of geology, such data include the spatial mapping of the geological layers and some disturbances. One should note that in engineering practice mostly simplified geology mapping in the form of a geoengineering cross-section is used. But on this basis only one- and two-dimensional simulations of the processes involved can be carried out. Whereas the spatial mapping of the geometry of an area enables three-dimensional analyses and numerical simulations, whereby the real conditions can be closely approximated. Thanks to this technique one can acquire the necessary geometry data and present and compare the analytical results in the space of the investigated area. A numerical terrain model was generated using the Bentley Systems CAD software suite (Bentley MicroStation V8i and the Bentley InRoads Group V8.11 application). The particular geological layers and the distinguished road pavement subareas characterized by varied geological parameters, obtained using GIS tools were implemented in the Flex PDE v.635 program. The geometry of the subgrade and the road with the generated finite elements is shown in Fig. 2.

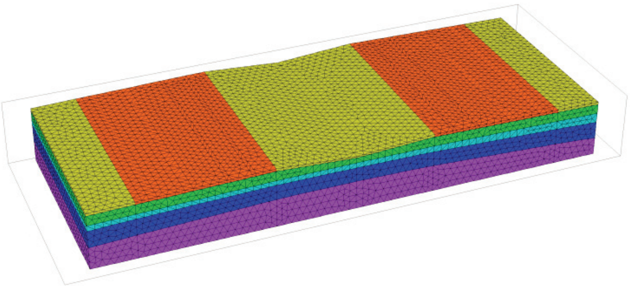


Fig. 2. Slice of finite element mesh generated by software

4.1. SIMULATION CONDITIONS

A simulation was run for the 200 m long section of the existing expressway, assuming load effect repeatability. Considering the differences between the layers in the road cross-section, the area was divided into 5 subareas (2 shoulders, 2 carriageways and the green central reservation). In this way, in total 25 subareas were created. The initial finite element mesh is shown in Fig. 2.

The displacements of the particular native soil layers and the road pavement courses under the dead weight and the temperature gradient between the lower surface of the deepest native soil layer and the surroundings were examined. Also the load generated by motor traffic was taken into account through the use of a band load imitating the running of the wheels of cars (fast lanes) and TIR lorries (crawler lanes). The total width of tyres amounting to 1.2 m and the load of 8.88 KN/m^2 were assumed for the TIR lorry. For the car the respective values were 0.3 m and 3.35 KN/m^2. The influence of temperature variation on the deformation of the whole area was analyzed. The experiment time of 20 years was assumed and the results were presented for the last 2 years of the simulation. The bottom surface of the investigated area was assumed to be permeable to heat ($T = 20$ °C) while the sides and top of the road area together with the native soil were defined as insulated. The horizontal components of the skeleton displacement vector on the top surface of the road were assumed to be equal to zero and the water pressure was assumed to be equal to the atmospheric pressure. Having analyzed the average monthly temperatures in the surroundings of the road area investigated, a sinusoid was fitted to the average temperature data through simple optimization on the basis of the minimization of the square error whereby boundary equation, describing the temperature variation over th

$T(t)=11$

The inf formation time was e

OF N
OF T

The Fl automatic fiable in t imposed c computatic

is the case here, another changeable element is time step dt. The program starts with time step $2*10^6$ s and ends with $2.63*10^6$. Figure 2 shows the distribution of finite elements in the initial stage of the computations.

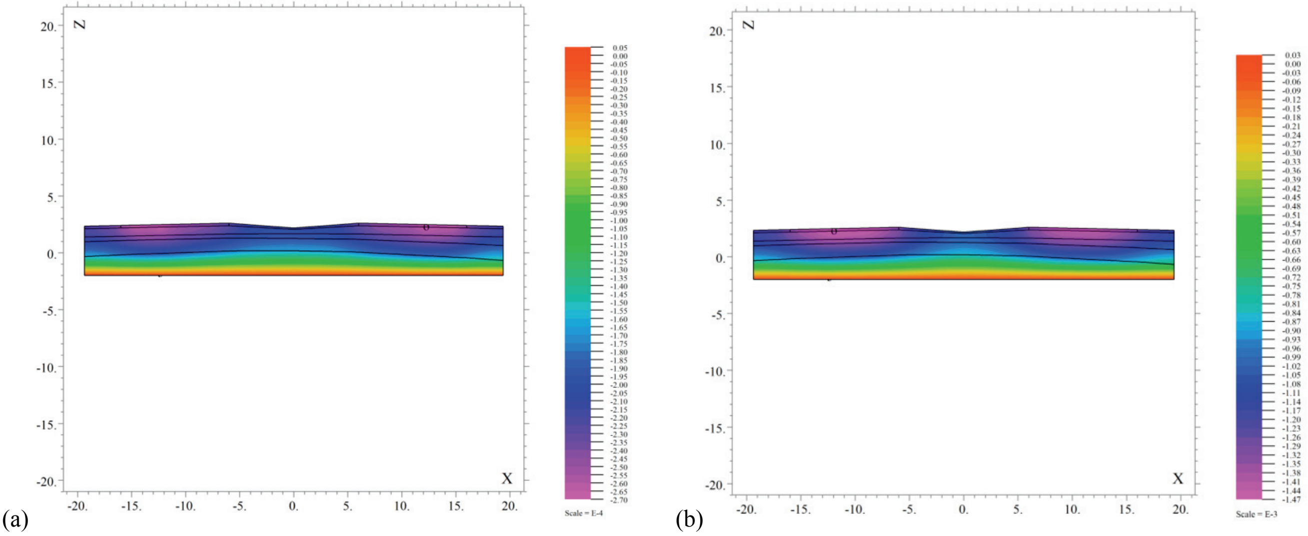
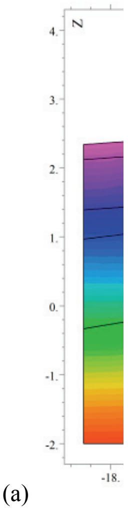


Fig. 5. Vertical displacements through the whole cross-section for: (a) June, (b) December



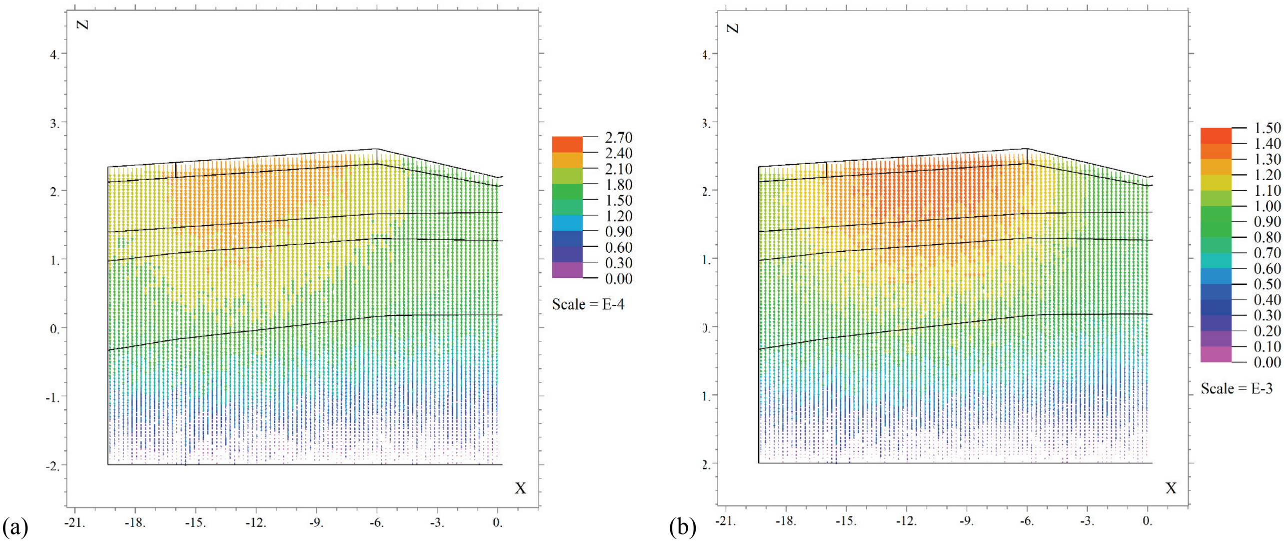
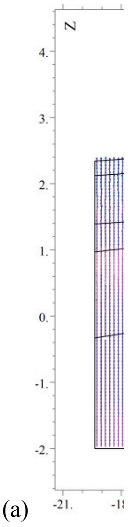


Fig. 6. Displacement vectors for half of the cross-section for: (a) June and (b) December



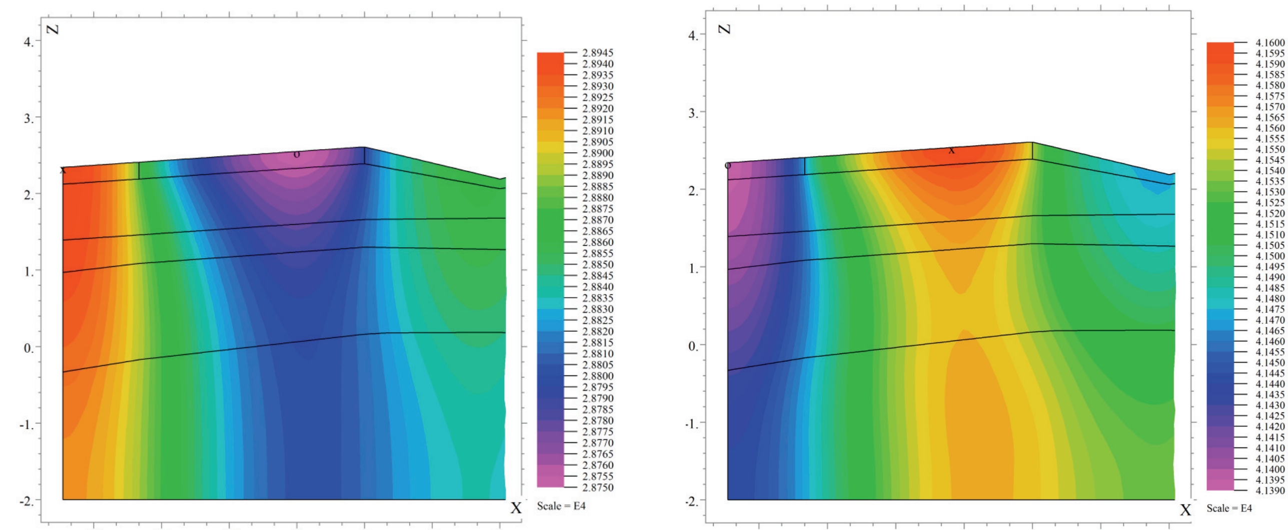


Fig. 7. Horizontal displacements for half of the cross-section in: (a) June and (b) December

Fig. 4. Heat flux: (a) June, (b) December

asphalt.

Figures 5a and 5b show the distribution of vertical displacements. The vectors of the displacements are shown in Figs. 6a and 6b. The largest displacements occur on the road surface in the central part of the two carriageways (in both the summer and the winter months). The displacements decrease with depth. Larger deformations (due to higher ambient temperatures) are observed in the summer months. One can also see (Fig. 5a) that the subsidence of the part of the carriageway on which TIR lorries drive is larger.

The stress diagram (Figs. 7a and 7b) shows that the highest stresses are induced in the winter months (Fig. 7b). In both cases, they are the highest on the outer shoulders and in the central part of the roadway.

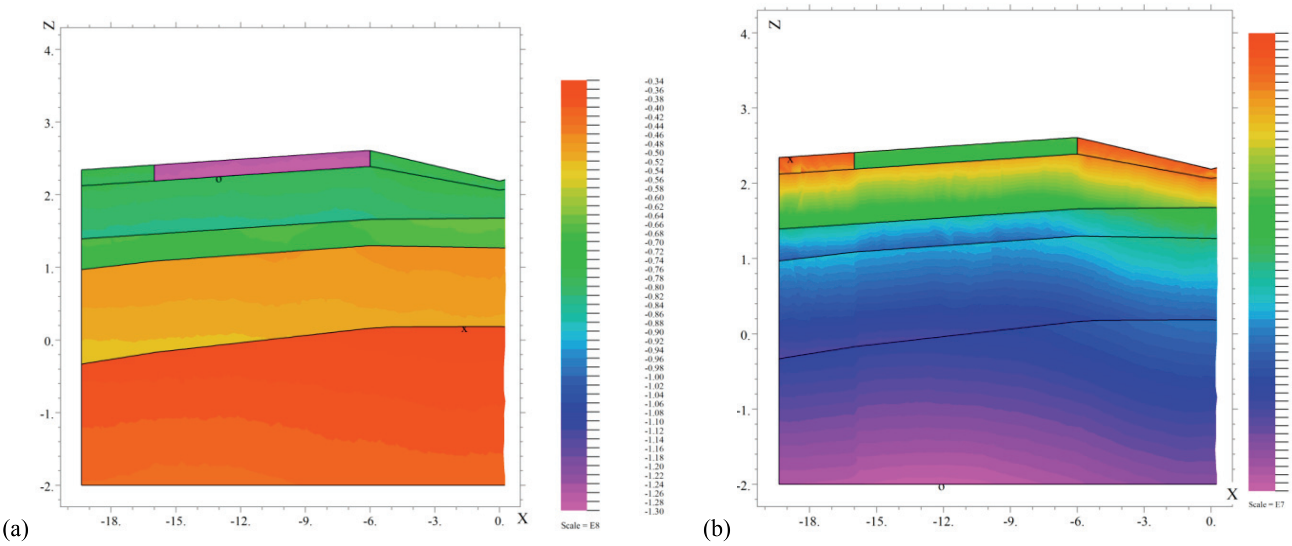
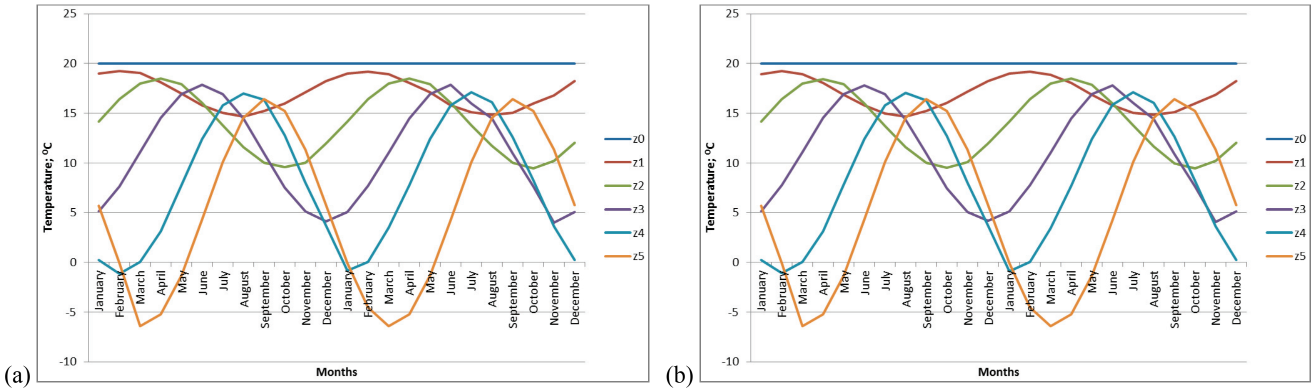


Fig. 8. Drucker–Prager potential for half of the cross-section in: (a) June and (b) December



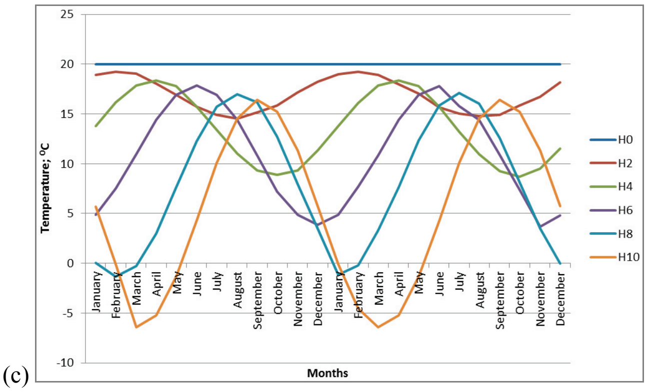


Fig. 9. Diagram of temperature [°C] at different depths *z* for:
(a) edge of roadway,
(b) middle of roadway, (c) inner part of roadway

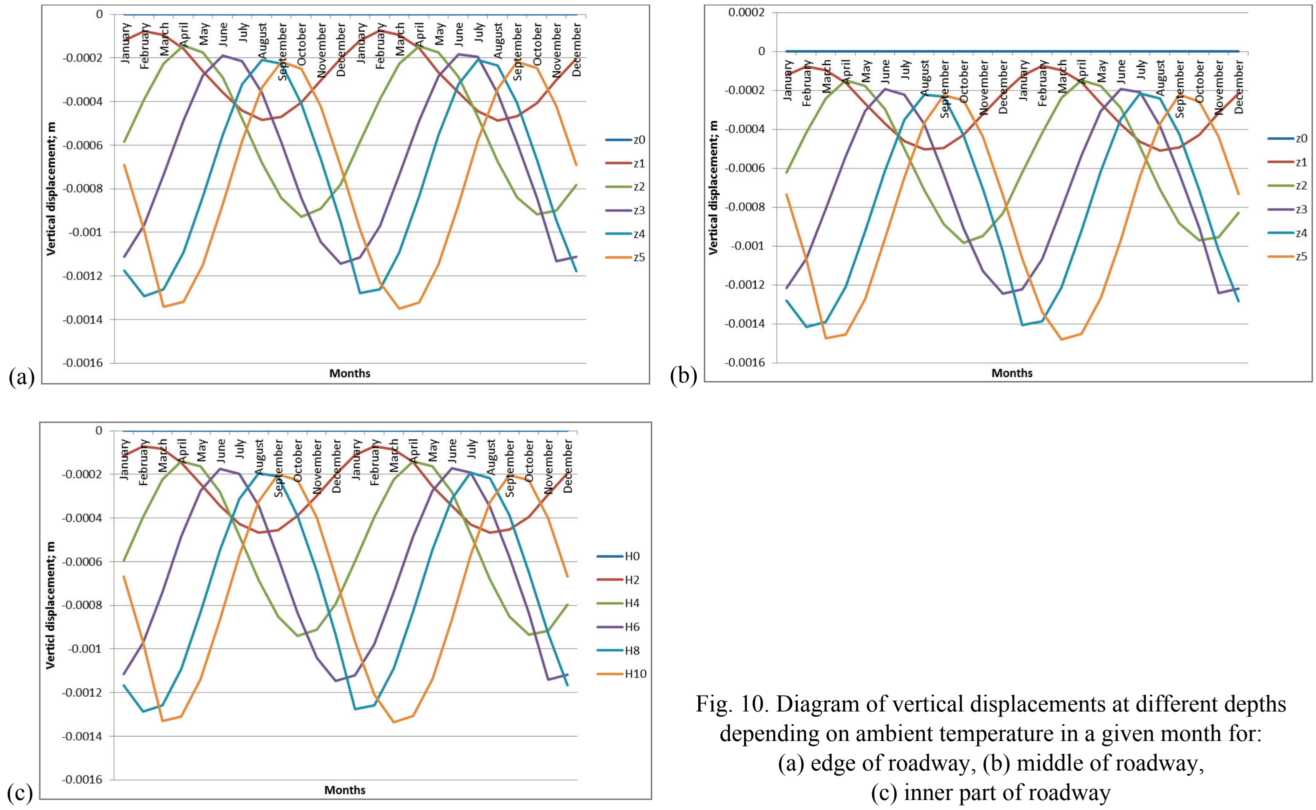


Fig. 10. Diagram of vertical displacements at different depths depending on ambient temperature in a given month for: (a) edge of roadway, (b) middle of roadway, (c) inner part of roadway

The Drucker–Prager potential (Figs. 8a and 8b) shows areas with a possible loss of stability where the potential assumes positive values. This is observed in December (Fig. 8b) for both the roadway shoulders.

The distribution of temperature at different depths of the road, modelled for three points (the left side, middle and right side of the roadway) shows that in all the cases (8a–c) the largest temperature fluctuations on the surface of the road occur in the summer months and they are closely correlated with the ambient temperature. The differences decrease with depth. The influence of traffic in this time scale is imperceptible.

Displacements versus temperature are shown in Fig. 9a–c. Depths H0, H2 … H10 are equally distributed along the total depth at a given point, i.e., H0 is the surface level, H10 is the domain bottom. The largest displacements occur on the surface of the road in the summer months and the smallest on its bottom in the winter months. The analysis confirmed the influence of temperature on the deformation of the roadway, but in this time no influence of vehicular traffic was observed.

6. CONCLUSIONS

The operation of roads is an important field of road engineering. It includes problems relating to the management, maintenance and necessary (because of road wear) renovations and alterations (to cope with increasing road traffic) of the roads. The constantly increasing traffic on the roads, the increased dynamic load resulting from accelerations and decelerations of vehicles and the increasing road transport have an adverse effect on the condition of the road pavement. Not only the building of roads, but also their maintenance has become one of the principal national economic-technical problems requiring both know-how and greater expenditures.

This paper presented the results of numerical computations aimed at determining the influence of temperature on the deformation of a two-phase medium described by the equations of Biot consolidation with the Kelvin–Voigt rheological skeleton, using as an example the thermal consolidation of the pavement of expressway S17 and taking into account the deformation of the native soil constituting the subgrade for the intensively used road. The 3D geometrical model of the area investigated was based on data obtained from the land surveying and soil investigation of a 200 m long section of the expressway and its shoulders. The present paper is the continuation of the numerical studies on the influence of temperature on the deformation process [10], [11]. The numerical solution assumed an analysis of the displacements of the particular layers of the road pavement and the native soil, subjected to the dead weight and the temperature gradient between the bottom surface of the native soil and the environment. The authors presented a three-dimensional model extended with the dynamic load of the modelled area. The experiment duration of 20 years was assumed and the results were presented for the last 2 years of the simulation.

The obtained distributions of temperatures and displacements in the native soil and on the surface of the modelled road, depending on the ambient temperature indicate the influence of the latter on the road deformation process. The studies show that the higher the ambient temperature, the larger the deformations both inside the road and on its surface. The highest variations in temperature occur in the road pavement upper courses and so the latter are the most prone to subsidence. The presented solution of the problem of creep over time for the intensively used expressway is another example of the practical application of the mathematical model of the consolidation of the Biot body with the Kelvin–Voigt rheological skeleton. The fact that the influence of temperature on the creep process is taken into account can significantly help to determine the areas being at risk of stability loss.

REFERENCES

[1] GRADKOWSKI K., *Load-bearing capacity of motor road pavements strengthened with geosynthetic materials*, Warsaw University of Technology Publishing House, Warsaw, 2008, (in Polish).

[2] BIOT M.A., *General theory of three-dimensional consolidation*, J. Appl. Phys., 1941, No. 12, 155.

[3] BIOT M.A., *General Solutions of the Equations of Elasticity and Consolidation of a Porous Material*, J. Appl. Mech., 1956, 23.

[4] BARTLEWSKA-URBAN M., STRZELECKI T., *Effect of variation of temperature field on the process of thermal consolidation of tailings pond "Żelazny Most"*, Studia Geotechnica et Mechanica, 2014, 36(2).

[5] KONDRACKI J., *Geograficzna mapa Polski*, Państ. Wydaw. Naukowe, 1978.

[6] BARTLEWSKA M., *Determination of the effective parameters of cohesive soil rheological models*, PhD thesis, Wrocław University of Technology, Faculty of Geoengineering, Mining and Geology, Wrocław, 2009, (in Polish).

[7] Geological Survey Plan, 2011.

[8] FlexPDE professional v 6.0 manual.

[9] BARTLEWSKA-URBAN M., STRZELECKI T., URBAN R., *Determination of effective parameters of Biot model with the Kelvin–Voight rheological skeleton,* Archives of Civil and Mechanical Engineering 01/2015. DOI: 10.1016/j.acme.2014.12.007.

[10] BARTLEWSKA M., STRZELECKI T., *Equations of Biot consolidation with Kelvin–Voigt rheological frame*, Studia Geotechnica et Mechanica, 2009, Vol. XXXI, No. 2, 3–15.

[11] BARTLEWSKA M., STRZELECKI T., *One-dimensional consolidation of the porous medium with the rheological Kelvin–Voight skeleton*, Studia Geotechnica et Mechanica, 2008, Vol. XXX, No. 1–2.