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Rodica BRANZEI*,
Marco DALL'AGLIO**

ALLOCATION RULES INCORPORATING INTERVAL UNCERTAINTY

This paper provides several answers to the question "How to cope with rationing problems with interval data?" Interval allocation rules which are efficient and reasonable are designed, with special attention to interval bankruptcy problems with standard claims and allocation rules incorporating the interval uncertainty of the estate.

Keywords: *allocation rules, bankruptcy, interval uncertainty*

1. Introduction

Various disputes including those generated by inheritance and bankruptcy (O'Neill [8]), firms' allocation of funds among their divisions (Pulido, Sánchez-Soriano and Llorca [10]), assignment of taxes (Young [13]), mass privatization of state-owned enterprises (Young [14]) are often affected by interval uncertainty regarding the homogeneous divisible good at stake. To deal with such situations, we design allocation rules incorporating interval uncertainty inspired by the existing literature on classical bankruptcy and taxation rules (Thomson [11]) and by rules for division problems under interval uncertainty regarding claims (Branzei et al. [4]).

To cope with interval uncertainty, we denote by $I(R_+)$ the set of all closed and bounded intervals in R_+ and by $I(R_+)^N$ the set of all n-dimensional vectors whose elements belong to $I(R_+)$. Let I, $J \in I(R_+)$ with $I = [\underline{I}, \overline{I}]$, $J = [\underline{J}, \overline{J}]$, then,

* Faculty of Computer Science, "Alexandru Ioan Cuza" University, Iaşi, Romania, e-mail: branzeir@info.uaic.ro

** Faculty of Economics, "Gabriele d'Annunzio" University, Chieti-Pescara, Italy, e-mail: marco.dallaglio@unich.it

$I+J=[\underline{I}+\underline{J},\overline{I}+\overline{J}]$. We say that I is weakly better than J, which we denote by $I \succeq J$, if and only if $\underline{I} \geq \underline{J}$ and $\overline{I} \geq \overline{J}$. We also use the reverse notation $J \preceq I$, meaning $I \succeq J$.

The remainder of this paper is organized as follows. Section 2 briefly presents bankruptcy problems and rules under three scenarios regarding interval uncertainty: uncertainty-free estate and claims; uncertainty free estate but interval uncertainty regarding claims; interval uncertainty regarding both the estate and claims. In Section 3 we focus on bankruptcy situations where only the estate is affected by interval uncertainty whereas the claims are more standard. We propose two families of interval rules incorporating the interval uncertainty regarding the estate which are based on some classical bankruptcy rules and show that they are efficient and reasonable. Section 4 deals with the general setting of bankruptcy problems with interval data. Here we propose efficient interval allocation rules which are based on some classical bankruptcy rules and procedures to transform claim intervals into more standard claims. It turns out that interval rules based on different procedures satisfy particular forms of reasonability. In Section 5 we place our work in the existing literature on bankruptcy problems with interval data.

2. Bankruptcy situations and rules

Let $N = \{1, 2, ..., n\}$ be a fixed set of claimants among which an estate has to be divided. A classical bankruptcy situation with a set of claimants N is a pair (E, c), where $E \geq 0$ is the estate to be divided and $c \in R_+^N$ is the vector of claims such that $\sum_{i \in N} c_i \geq E$. We denote by BR^N the set of bankruptcy situations (E, c) with player set N. The total claim is denoted by $C = \sum_{i \in N} c_i$. A bankruptcy rule is a function $f: BR^N \to R_+^N$ which assigns to each bankruptcy situation $(E,c) \in BR^N$ a payoff vector $f(E,c) \in R_+^N$ such that $0 \leq f(E, c) \leq c$ (reasonability) and $\sum_{i \in N} f_i(E,c) = E$ (efficiency). In this paper we are interested in bankruptcy rules that are continuous and coordinate-wise (weakly) increasing in E. We recall the most widely used rules. The proportional (*PROP*) rule assigns awards proportional to claims, i.e. $PROP_i(E,c) = \frac{c_i}{C} E$ for all $i \in N$. The constrained equal awards rule (*CEA*) assigns equal amounts to all claimants subject to no one receiving more than his/her claim, and is defined by $CEA_i(E, c) = \min\{c_i, \alpha\}$ for all $i \in N$, where α is determined by $\sum_{i \in N} CEA_i(E,c) = E$. The constrained equal losses rule (*CEL*) determines equal losses (the value of the claim unsatisfied) subject to no one receiving a negative amount, and is defined by $CEL_i(E, c) = \max\{c_i - \beta, 0\}$ for all $i \in N$, where β is determined by $\sum_{i\in N} CEL_i(E,c) = E$. The Talmud rule (*TAL*, Aumann and Maschler [1]) combines the ideas of *CEA* and *CEL*. Its value, $TAL(E, c)$, is given by $CEA\left(E, \frac{c}{2}\right)$ if $E \le \frac{c}{2}$ and by $\frac{c}{2} + CEL\left(E - \frac{c}{2}, \frac{c}{2}\right)$ otherwise. The random arrival rule (*RA*, O'Neill [8]) assumes that claimants are fully compensated, one after the other, until the money runs out. The order of arrival is random, and the rule considers the average compensation of each claimant. Consequently, the definition is given by $RA_i(E,c) = \frac{1}{n!}\sum_{\pi\in\Pi^N} \min\left\{c_i, \max\left\{E - \sum_{j\in N, \pi(j)<\pi(i)} c_j, 0\right\}\right\}$ for all $i \in N$, where Π^N is the class of bijections from N into itself. Finally, the adjusted proportional rule (*AP*, Curiel, Maschler and Tijs [6]) refines the proportional method by assigning to each claimant the amount conceded to him/her by the others (his/her minimal right) plus a share of the remainder of the estate proportional to the remainders of the claims, which are truncated with respect to the remainder of the estate, i.e.

$$AP(E,c) = m(E,c) + PROP\left(E - \sum_{j\in N} m_j(E,c), \left(\min\left\{c_i - m_i(E,c), E - \sum_{j\in N} m_j(E,c)\right\}\right)_{i\in N}\right),$$

with $m_i(E, c)$ being the minimal right of claimant i, $i \in N$, and $m(E, c) = (m_i(E, c))_{i\in N}$.

The last three rules are particular solutions of the bankruptcy game $v_{(E,c)}$, which compute the minimal rights for each coalition S, $\varnothing \neq S \subset N$, as $\min v_{(E,c)}(S) = \min\left\{E - \sum_{i\in N\setminus S} c_i, 0.\right\}$. In fact, if we consider the Shapley value, the prenucleolus and the τ-value of $v_{(E,c)}$, we recognize the random arrival rule, the Talmud rule, and the adjusted proportional rule, respectively, for the classical bankruptcy situation (E, c).

Situations where the estate E is uncertainty-free but claimants are facing uncertainty regarding their effective rights, knowing only the lower and upper bounds on their claims, have already been examined. In this context, Branzei et al. [4] described two types of families of allocation rules leading to efficient and reasonable solutions for bankruptcy situations with interval claims: one-stage rules based on compromise claims and multi-stage rules based on compromise claims in adjusted claim intervals. In Section 3, inspired by some of these rules, we design allocation rules for bankruptcy situations where interval uncertainty affects the estate as well as the claims.

An interval bankruptcy situation with set of claimants N is a pair (E, c), where $E = [\underline{E}, \overline{E}]$ is the uncertain estate to be divided and $c \in I(R)^N$ is the vector of interval claims $([\underline{c}_1, \overline{c}_1], ..., [\underline{c}_n, \overline{c}_n])$ such that $\overline{E} \le \underline{C}$, where $\underline{C} = \sum_{i\in N} \underline{c}_i$ (Branzei and Alparslan Gök [2]). We denote by BRI^N the family of bankruptcy interval situations with player set N. Note that for each bankruptcy situation $(E, c) \in BRI^N$ all "selections" $(\widetilde{E}, \widetilde{c})$, where $\widetilde{E} \in [\underline{E}, \overline{E}]$ and $\widetilde{c} = (\widetilde{c}_i)_{i \in N}$ with $\widetilde{c}_i \in [\underline{c}_i, \overline{c}_i]$ for each $i \in N$, are elements of BR^N because $\widetilde{C} \geq \underline{C} \geq \overline{E} \geq \widetilde{E}$, where $\widetilde{C}$ is the total selection claim.

A bankruptcy interval rule is a function $F : BRI^N \to I(R_+)^N$ assigning to each bankruptcy situation $(E, c) \in BRI^N$ a vector $F(E,c) \in I(R_+)^N$ such that:

- $[0,0] \preceq F_i(E,c) \preceq [\underline{c}_i, \overline{c}_i]$ for each $i \in N$ (reasonability);
- $\sum_{i=1}^{n} F_i(E,c) = [\underline{E}, \overline{E}]$ (efficiency).

We call a rule weakly reasonable if $[0,0] \preceq F_i(E,c) \preceq [\underline{c}_i, \overline{c}_i]$ for each $i \in N$.

3. Allocation rules incorporating the interval uncertainty regarding the estate

In this section we focus on the subclass of interval bankruptcy situations where the estate is affected by interval uncertainty, whereas all the claims are given in the form of degenerate intervals, i.e. $\underline{c}_i = \overline{c}_i$ for all $i \in N$. For simplicity, we denote the vector of claims by $c = (c_i)_{i \in N}$, where $c_i \in R_+$ is the standard claim of i, and by $BRDC^N$ the related set of bankruptcy problems (E, c) where $E \in [\underline{E}, \overline{E}]$ with $\overline{E} > \underline{E}$.

In the sequel, we propose two families of interval allocation rules which are based on classical bankruptcy rules f which are continuous and coordinate-wise (weakly) increasing with respect to the estate. Most bankruptcy rules possess this property. In our setting this implies that for all $E_1, E_2 \in [\underline{E}, \overline{E}]$ with $E_1 < E_2$ it follows that $f_i(E_1, c) \leq f_i(E_2, c)$ for all $i \in N$. First, we introduce the interval allocation rule based on f, $F^f : BRDC^N \to I(R_+)^N$, defined by $F_i^f(E,c) = [f_i(\underline{E},c), f_i(\overline{E},c)]$, for all $(E, c) \in BRDC^N$ and $i \in N$.

Proposition 1. The interval allocation rule F^f is efficient and reasonable.

Proof: Let $(E, c) \in BRDC^N$. Notice that the coordinate-wise (weakly) increasing property of f with respect to the estate guarantees that $f_i(\underline{E},c) \leq f_i(\overline{E},c)$ for all $i \in N$. The efficiency of F^f follows from the efficiency of f. Furthermore, by the reasonability of f we obtain, for each $i \in N$,

$$[0,0] \preceq F_i^f(E,c) \preceq [c,c].$$

□

Next, we introduce another family of interval allocation rules in $BRDC^N$ based on classical bankruptcy rules f which satisfy the same assumptions as before. Here, we use a two-stage approach:

Stage 1. The lower bound on the value of the estate, $\underline{E}$, is distributed over the claimants based on f by using the initial claims c_i, $i \in N$. Denote by $f_i(\underline{E},c)$ the share obtained by individual i for $i \in N$.

Stage 2. The amount $\overline{E}-\underline{E}$ is distributed over the claimants based on f by using the residual claims $c_i - f_i(\underline{E},c)$, $i \in N$.

Given f, we introduce the two-stage interval allocation rule based on f, $F^{f,(2)}$: $BRDC^N \to (R_+)^N$, defined by

$$F_i^{f,(2)}(E,c) = [f_i(\underline{E},c)\ f_i(\underline{E},c) + f_i(\overline{E}-\underline{E}, c - f(\underline{E},c))]$$

for all $i \in N$ and $(E, c) \in BRDC^N$.

Proposition 2. The interval allocation rule $F^{f,(2)}$ is efficient and reasonable.

Proof: The efficiency of $F^{f,(2)}$ is obvious. To prove the reasonability we note first that from the reasonability of f we have $0 \le f_i(\underline{E},c) \le c_i$, and also

$$0 \le f_i(\underline{E},c) + f_i(\overline{E}-\underline{E}, c - f(\underline{E},c)) \le f_i(\underline{E},c) + c_i - f_i(\underline{E},c) = c_i$$

for each $i \in N$.

□

4. Allocation rules incorporating the uncertainty regarding the estate and the claims

In this section we introduce some families of interval allocation rules in the class BRI^N also based on well known classical bankruptcy rules f which are continuous and coordinate-wise (weakly) increasing with respect to the estate. Notice that in such a bankruptcy problem $(E, c) \in BRI^N$ all the data (the estate and the claims) are affected by interval uncertainty. Let f be the classical bankruptcy rule agreed upon. We try to overcome the extra difficulty introduced by the interval uncertainty regarding claims with the aid of compromise factors $t_i \in [0, 1]$, $i \in N$. Let t_i be the compromise factor proposed by claimant i for transforming his/her interval claim $[\underline{c}_i, \overline{c}_i]$ into the more standard claim $c_i^{t_i} = t_i\overline{c}_i + (1-t_i)\underline{c}_i$ for each $i \in N$.

We introduce the *t-compromise interval allocation rule* based on f, $F^{t,f}$: $BRI^N \to (R_+)^N$, defined by

$$F_i^{t,f}(E,c) = [f_i(\underline{E},c^t), f_i(\overline{E},c^t)] \text{ for each } i \in N.$$

We call such a rule *t-reasonable* if $[0, 0] \preceq F_i^{t,f}(E,c) \preceq [c_i^{t_i}, c_i^{t_i}]$ for each $i \in N$. Since $c_i^{t_i} \le \overline{c}_i$ for each $i \in N$, t-reasonability implies weak reasonability.

Proposition 3. The interval allocation rule $F^{t,f}$ is efficient and t-reasonable.

Proof: Let $t = (t_1, ..., t_n) \in [0, 1]^N$. The efficiency of $F^{t,f}$ follows from the efficiency of rule f. The t-reasonability of $F^{t,f}$ follows from $0 \le f_i(\underline{E},c^t) \le c_i^{t_i}$ and $0 \le f_i(\overline{E},c^t) \le c_i^{t_i}$ for each $i \in N$.

□

We notice that the vector of compromise factors $t \in [0, 1]^N$ plays a key role in defining the family of interval allocation rules $F^{t,f}$. Another scenario for removing the uncertainty regarding claims is by compromising the lower and upper claims in a uniform way, using the same $t \in [0, 1]$, which leads to $c^t = (c_i^{t_i})_{i \in N}$, with $c_i^t = t\overline{c}_i$ + $(1-t)\underline{c}_i$ $(1 - t)$ for each $i \in N$. The choice of such a compromise factor t could be made by a neutral arbitrator.

A more sophisticated procedure to introduce interval allocation rules, based on the idea of compromising the individual lower and upper claims, is to average t-compromise solutions according to a given probability measure (Branzei et al. [4]). Let μ be a probability measure on $\langle [0,1], \mathfrak{B} \rangle$, where $\mathfrak{B}$ is the σ-algebra of Borel subsets of [0, 1]. Common examples of such probability measures are the Lebesgue measure λ and the Dirac measure δ_a, with $a \in [0, 1]$ being an atom. Consider the t-compromise bankruptcy situations $(\underline{E},c^t)$ and $(\overline{E},c^t)$ for $t \in [0, 1]$. We introduce the μ-compromise interval allocation rule based on f, $F^{f,\mu}$: $BRI^N \to I(R_+)^N$, defined by

$$F_i^{f,\mu}(E,c) := \left[\int_0^1 f_i(\underline{E},c^t) d\mu(t), \int_0^1 f_i(\overline{E},c^t) d\mu(t) \right]$$

for all $(E, c) \in BRI^N$ and $i \in N$.

Proposition 4. The interval allocation rule $F^{f,\mu}$ is efficient and weakly reasonable.

Proof: First, from the coordinate-wise monotonicity of f with respect to the estate, we have $f_i(\underline{E},c^t) \le f_i(\overline{E},c^t)$ for each $i \in N$. By integrating over [0, 1] and using the monotonicity property of integrals we obtain, for all $i \in N$,

$$\int_0^1 f_i(\underline{E},c^t)d\mu(t) \le \int_0^1 f_i(\overline{E},c^t)d\mu(t)\,.$$

To derive the efficiency of $F^{f,\mu}$, consider the following relations:

$$\sum_{i\in N}\int_0^1 f_i(\underline{E},c^t)d\mu(t) = \int_0^1 \sum_{i\in N} f_i(\overline{E},c^t)d\mu(t) = \int_0^1 \underline{E}d\mu(t) = \underline{E}\,;$$

$$\sum_{i\in N}\int_0^1 f_i(\overline{E},c^t)d\mu(t) = \int_0^1 \sum_{i\in N} f_i(\overline{E},c^t)d\mu(t) = \int_0^1 \overline{E}d\mu(t) = \overline{E}\,.$$

Furthermore, from $0 \le f_i(\underline{E},c^t) \le f_i(\overline{E},c^t) \le c_i^t \le \overline{c}_i$ for all $i \in N$, by integrating over [0, 1] and using the monotonicity property of integrals, we obtain

$$[0,0] \preceq F^{t,\mu}(E,c) \preceq \left[\int_0^1 c_i^t d\mu(t), \int_0^1 c_i^t d\mu(t)\right] \preceq [\overline{c}_i, \overline{c}_i]\,.$$

□

Now, we define a family of two-stage interval allocation rules based on any classical bankruptcy rule *f* as follows. In the first stage we distribute the lower bound on the value of the estate $\underline{E}$ according to the lower bounds on the claims. In the second stage we distribute $\overline{E} - \underline{E}$ according to the residual values of the claims based on the vector of allocations determined in the first stage. Given *f*, we introduce the two-stage allocation rule based on *f*, $G_i^{f,(2)} : BRI^N \to I(R_+)^N$, defined by

$$G_i^{f,(2)}(E,c) = [f_i(\underline{E},c), f_i(\underline{E},\underline{c}) + f_i(\overline{E} - \underline{E}, \overline{c} - f(\underline{E},\underline{c}))]\,,$$

for all $i \in N$ and $(E, c) \in BRI^N$.

Proposition 5. The two-stage interval allocation rule $G^{f,(2)}$ is efficient and reasonable.

Proof: The efficiency of $G^{f,(2)}$ is obvious. Furthermore, $0 \le f_i(\underline{E},c) \le c_i$ for all $i \in N$, and

$$0 \le f_i(\underline{E},c) + f_i(\overline{E} - \underline{E}, \overline{c} - f(\underline{E},\underline{c})) \le f_i(\underline{E},c) + \overline{c} - f_i(\underline{E},\underline{c}) = \overline{c}_i\,,$$

for all $i \in N$.

□

We notice that our two-stage approach to the interval setting is related to the “minimal rights first” and “composition up” properties of classical bankruptcy rules.

The "minimal rights first" property requires the rule to first attribute the minimal right of each creditor, i.e. the maximum of zero and the difference between the sum of the claims of all the other creditors and the estate; then to distribute the remainder over the claimants by taking into account the revised claims obtained by reducing each individual claim by the corresponding minimal right. For bankruptcy situations with reference points (Pulido, Sánchez-Soriano and Llorca [10], Pulido et al. [9]), the exogenously given reference point for the allocation of the estate can naturally play the role of a minimal rights allocation in the case where the estate is sufficient to give each player his/her reference amount. Furthermore, since bankruptcy situations with reference points can be naturally embedded in the class of interval bankruptcy situations whose estate is known with certainty, compromise rules developed for CERO- and CREO-bankruptcy situations (Pulido et al. [9]) become good candidates for the resolution of conflicting interval claims. The "composition up" property is related to the occurrence of an excess of the estate after solving the bankruptcy problem, and requires the rule of allowing creditors to keep their initial awards, revise their claims down by these awards, and divide the residual amount according to these revised claims. We refer the reader to Yeh [12] for an interesting study of these two interesting properties in connection with the "secured lower bound" property within the context of classical bankruptcy situations (Moreno-Ternero and Villar [7]).

5. Concluding remarks

This paper considers bankruptcy situations with interval data. Some families of efficient and reasonable interval bankruptcy rules are introduced with the aid of classical bankruptcy rules which are continuous and coordinate-wise (weakly) increasing with respect to the estate. Other interval rules related to bankruptcy problems with interval data have already been introduced in the recent literature. In Branzei and Alparslan Gök [2] the interval proportional rule and the interval rights-egalitarian rule are introduced on suitable subclasses of $BRIG^N$ and related to the cores of cooperative interval games arising from the corresponding interval bankruptcy situations. In Branzei, Dall'Aglio and Tijs [3] interval bankruptcy rules which are interesting from a game-theoretic point of view are introduced and studied. The common characteristic of all interval bankruptcy rules is that such rules provide a vector of intervals which is useful at en ex-ante stage to inform claimants what they can expect, between two boundaries, from the interval bankruptcy problem in question. The interval allocation generated by an interval bankruptcy rule will be transformed into a standard allocation when the realizations of the estate and/or the claims occur at an ex-post stage. To transform a vector of closed and bounded intervals into a standard vector is not a trivial task (Branzei, Tijs and Alparslan Gök [5]). However, in the case where vectors of intervals are obtained via interval rules introduced in this paper, the derivation of a standard allocation corresponding to the realization $\breve{E}$ of the value of the estate and realizations $(\breve{c}_i)_{i \in N}$ of the claims is quite easy from a computational point of view. We only need to apply the chosen rule f to the classical bankruptcy problem $(\breve{E}, \breve{c})$ for any of the one-stage rules, whereas standard allocations $(x_1, ..., x_n)$ corresponding to two-stage rules are given by $x_i = f_i(\underline{E}, \underline{c}) + f_i(\breve{E} - \underline{E}, \breve{c} - f(\underline{E}, \underline{c}))$ for each $i \in N$. Notice that the properties of f ensure that the obtained allocation $(x_1, ..., x_n)$ belongs to the interval allocation $(I_1, ..., I_n)$ in question, i.e. $x_i \in I_i$ for all $i \in N$.

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References

[1] AUMANN R.J., MASCHLER M., *Game theoretic analysis of a bankruptcy problem from the Talmud*, Journal of Economic Theory, 1985, 36, 195–213.

[2] BRANZEI R., ALPARSLAN GÖK S.Z., *Bankruptcy problems with interval uncertainty*, Economics Bulletin, 2008, Vol. 3, No. 56, 1–10.

[3] BRANZEI R., DALL'AGLIO M., TIJS S., *Interval game theoretic division rules*, CentER Discussion, 2008, Paper No. 97.

[4] BRANZEI R., DIMITROV D., PICKL S., TIJS S., *How to cope with division problems under interval uncertainty of claims*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 2004, 12, 191–200.

[5] BRANZEI R., TIJS S., ALPARSLAN GÖK S.Z., *How to handle interval solutions for cooperative interval games*, preprint No. 110, Institute of Applied Mathematics, METU, 2008.

[6] CURIEL I.J., MASCHLER M., TIJS S.H., *Bankruptcy games*, Zeitschrift für Operations Research, 1987, 31, A143–A159.

[7] MORENO-TERRERO J.D., VILLAR A., *The Talmud rule and the securement of creditors' awards*, Mathematical Social Sciences, 2004, 47, 245–257.

[8] O'NEILL B., *A problem of rights arbitration from the Talmud*, Mathematical Social Sciences, 1982, 2, 345–371.

[9] PULIDO M., BORM P., HENDRICKX R., LLORCA N., SÁNCHEZ -SORIANO J., *Compromise solutions for bankruptcy situations with references*, Annals of Operations Research, 2008, 158, 133–142.

[10] PULIDO M., SÁNCHEZ -SORIANO J., LLORCA N., *Game theoretic techniques for university management: an extended bankruptcy model*, Annals of Operations Research, 2002, 109, 129–142.

[11] THOMSON W., *Axiomatic and game theoretic analysis of bankruptcy and taxation problems : a survey*, Mathematical Social Sciences, 2003, 45, 249–297.

[12] YEH C.-H., *Secured lower bound, composition up, and minimal rights first for bankruptcy problems*, Journal of Mathematical Economics, 2008, 44, 925–932.

[13] YOUNG H.P., *On dividing an amount according to individual claims or liabilities*, Mathematics of Operations Research, 1987, 12, 398–414.
[14] YOUNG H.P., *Distributive justice in taxation*, Journal of Economic Theory, 1988, 48, 321–335.

Reguły alokacji z niepewnością przedziałową

W pracy przedstawiono szereg odpowiedzi na pytanie Jak radzić sobie z racjonalizacją problemów związanych z danymi przedziałowymi? Zaprojektowano efektywne i uzasadnione przedziałowe zasady alokacji, w celu informowania ludzi lub podmiotów gospodarczych, stających wobec przedziałowej niepewności, o „rzetelnych" dolnej i górnej granicach ich osiągalnych podziałach akcji. Po zaprezentowaniu problemów związanych z bankructwem i prezentacji różnych zasad postępowania w szeregu scenariuszy związanych z przedziałową niepewnością, skupiliśmy swoją uwagę na sytuacjach, kiedy przedziałowa niepewność odnosi się tylko do majątku firmy, podczas gdy pozostałe żądania pozostają standardowymi. Proponujemy dwie rodziny reguł przedziałowych, stosujących przedziałową niepewność w opisie majątku i bazujących na klasycznych regułach bankructwa oraz pokazujących swoją efektywność i racjonalność. W bardziej ogólnych przypadkach, w których zawarte są przedziałowe żądania, także proponujemy efektywne reguły alokacji przedziałowej, bazującej na klasycznych regułach bankructwa i procedurach transformacji żądań przedziałowych w żądania bardziej klasyczne. Stwierdzamy, że reguły przedziałowe bazujące na różnych procedurach spełniają szczególne postulaty racjonalności. Nasze osiągnięcia umiejscawiamy na tle literatury poświęconej problematyce bankructw z danymi przedziałowymi.

Słowa kluczowe: *zasady alokacji, bankructwo, przedziałowa niepewność*