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USE OF GRANULANDS FOR ANALYSIS OF SOCIAL CLASS

In this paper, an analytical tool enabling the analysis of social stratification is proposed. The classical scheme for scaling consisting of two stages, conceptualisation and operationalization, is modified by the use of the concept of granulation introduced by L. Zadeh. The essential step in the modified scheme for the quantification of vague concepts concerning social class is realized using linguistic variables. The essential part of the methodology presented is illustrated by a simple hypothetical example. However, the methodology is suitable for any classification problem when classes are defined verbally.

Keywords: *granuland, social stratification, fuzzy sets, linguistic variables, social standing, social class, fuzzy classification*

1. Introduction

It is well known that Zadeh's motivation for introducing the theory of fuzzy sets was to facilitate the modelling and analysis of so-called humanistic systems. Zadeh himself defined humanistic systems as such systems in which human judgment, perception and emotive reasoning play a substantive and important role.

Surprisingly enough, this theory has found clear applications in the seemingly exact fields of engineering and automatic control.

Within humanistic systems, medicine was the only discipline in which the possibility of applying fuzzy sets was recognized early on. To diagnose a disease, the composition of a fuzzy set of symptoms with a fuzzy relation representing medical knowledge relating symptoms to diseases is used.

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The most typical humanistic system, and the most fundamental one, is the system named “society”. Society is the most important social institution in every civilization. It is a major subject of study within social policy, sociology, social psychology, political science etc. It is strange and surprising, that fuzzy sets have hardly been applied at all to this system.

2. Social stratification

All people are born equal. Nonetheless, some are treated as noble while others are not, some have power, some are rulers, and others have to follow their orders.

There is division and stratification among people.

The study of socio-economic stratification has a long and complex history in the social sciences.

The central sociological issue is social stratification (see [17]). A social structure consists of different parts of a society, and these parts are interrelated. A social structure can be defined as a multidimensional space of different social positions among which a population is distributed (see [4]). Consequently the clearest task of sociology is the analysis of various forms of diversification. Social diversification is nothing more than an assignment of individual people to social positions. Blau distinguishes two basic forms of diversification: heterogeneity and inequality.

A social structure is delineated by its structural parameters, which are the axes of a multidimensional space of social positions.

However, the basic components of a social structure have no sharp borders, they are conceptualised as being certain groups or classes, but by using vague concepts

The objective of this article is not to discuss these questions, but to propose an analytical tool enabling the analysis of such problems using a quantitative methodology.

According to Weber there are three important dimensions of stratification [3]:

1) economic resources,

2) prestige

3) power.

Currently, mainly due to Duncan’s work, the following three dimensions are used [9]:

1) occupation,

2) education,

3) and income.

Some other authors add other characteristics such as value systems, lifestyle, information, etc. to these dimensions

In this article we do not restrict ourselves to any fixed set of dimensions. Instead, we present a general methodology enabling the consideration of any number of characteristics.

3. Background to the Problem

Suppose that there is a society consisting of N individuals $S = \{s_1, s_2, \ldots s_N\}$ divided into a number of classes. To keep the disussion simple enough, we confine ourself to three classes: lower, middle, and upper class.

For example, US society is considered to be composed of an upper class (16% of the population), middle class (64% of the population), and lower class (20% of the population).

If we assume that classes are defined nominally by some characteristics, then we can easily conclude that classes are not defined with sharp boundaries. Each person can belong to every class, with a different degree of membership.

Therefore, the problem is to define the appropriate membership functions, and then to determine the degrees of membership for each person to each class.

Let us denote the social classes by symbols $Sc1$, $Sc2$, ...,Sck. The degrees of membership of individual i to these classes will be denoted by the symbols:

$$\mu_{sc1}(s_i), \mu_{sc2}(s_i), \ldots, \mu_{sck}(s_i).$$

To solve this problem, it is assumed that each member of the society S is characterized by a set of n attributes, $Atr = \{A_1, A_2, \ldots, A_n\}$. For example, A_1 might stand for "age", A_2 for "income", A_3 for "family size" etc. A mathematical term "variable", denoted by X, is associated with each attribute A. This variable is treated as a measurement, or scaling, of attribute A, and therefore considered as a mapping:

$$\mathrm{X} : \mathrm{S} \to \mathrm{R}.$$

The variables $X_1, \mathrm{X}_2, \ldots, \mathrm{X}_\mathrm{n}$, corresponding to attributes A_1, A_2, ..., A_n, are called the dimensions or structural parameters of the social structure.

The value of attribute A_j for individual i is denoted as x_{ij}.

These measurements form the matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & & & \\ x_{m1} & x_{n2} & \cdots & x_{mn} \end{bmatrix}.$$

The measurements of all the attributes for the i-th individual are represented by the vector

$$x^i = (x_{i1}, x_{i2}, \ldots, x_{in}).$$

If the individual is fixed, or considered as a generic case, the index i will be omitted and the following simplified notation used

$$x = (x_1, x_2, ..., x_n).$$

For the ordered values, we will use the standard notation

$$x_{(1)} \le x_{(2)} \le ... \le x_{(n)}.$$

4. A granuland of the parameters of social structure

A granuland of the social structure is defined as a triple (see also [14]):

$$(2^{Atr}, G, h),$$

where

2^{Atr} is a measurable space of attributes,

G is a family of granulands for each parameter and each social class,

h is a vector of indicators of social standing.

Each of these notions will be defined.

First measurability and integration are defined.

A monotone measure of subsets of attributes is defined to be a function

$$m : 2^{Atr} \to [0, 1]$$

satisfying the following properties (see [5], [15], [27]):

1) $m(\phi) = 0$
2) $m(Atr) = 1$
3) $A \subset B \Rightarrow m(A) \le m(B), \quad A, B \in Atr$.

Having defined such a measure, one can define the inegral of the function h defined on the set of attributes with respect to this measure.

Here, we will consider two concepts of integrals with respect to such a monotone measure and the Weber integral with respect to a $\perp$ decomposable measure. The first integral used in this paper is the Sugeno integral, the second one is the Choquet integral.

The Sugeno integral is defined as follows:

$$\int_A h \circ m = \vee(t \wedge m(h > t)),$$

where $\vee$ and $\wedge$ denote the operations "sup" and "inf", respectively.

The Choquet integral is defined by the following formula (see [21]):

$$\int \mathrm{m}(\{x : h(x) > t\}\mathrm{d}t.$$

The Weber integral is based on the notion of a $\perp$ decomposable measure.

For the definition of a $\perp$ decomposable measure, the notion of a t-norm and t-conorm is needed.

Suppose that

$$g : [0,1] \to [0,\infty]$$

is an increasing and continuous function such that $g(0) = 0$.

The t-conorm of g, denoted by the symbol $\perp$, is defined as follows

$$a \perp b = g^{(-1)}(g(a) + g(b)),$$

where $g^{(-1)}$ is the pseudoinverse of g, defined by

$$g^{(-1)}(y) = \begin{cases} g^{(-1)}(y), & \text{if } y \in [0, g(1)], \\ 1, & \text{if } y \in [g(1), \infty]. \end{cases}$$

A $\perp$ decomposable measure is a function

$$m : 2^{Atr} \to [0,1]$$

satisfying the properties

1) $m(\phi) = 0$
2) $m(Atr) = 1$
3) $m(A \cup B) = m(A) \perp m(B), \quad A \cap B = \phi$

Weber's integral of h over A is defined by (see[21]):

$$\int_A h \perp m = g^{-1}\left(\int_A hd(m \circ g) \right).$$

Granulands will now be defined.

We associate $a(j)$ linguistic values denoted by symbols λ_{j1}, λ_{j2}, ..., $\lambda_{ja(j)}$ with each attribute A_j. This means that the attribute A_j is transformed into a linguistic variable (see [20], [21], [22]): ${}_*A_j = \{\lambda_{j1}, \lambda_{j2}, ..., \lambda_{ja(j)}\}$.

This linguistic variable is also called a starred variable, and the operation of transformation is called granulation [23].

Linguistic modifiers and operators are defined on the set ${}_*A_j$. The symbol ${}_*\overline{A_j}$ denotes the closure of ${}_*A_j$ with respect to these operators and modifiers. This means that ${}_*\overline{A_j}$ contains all the linguistic values which can be obtained from the base elements ${}_*A_j$ by using admissible linguistic operators and modifiers.

If, for example, A_j stands for the attribute “income”, then λ_{j1} might be “low”, λ_{j2} – “moderate”, λ_{j3} – “high”. The elements of ${}_*\overline{A_j}$ could be such as “not very low”, “very high”, “not high”, etc.

Each linguistic value from the set ${}_*A_j$ is defined by the appropriate membership function

$$\mu_{\lambda_{jk}} : \mathrm{C_j} \to [0,1] ,$$

where C_j is a codomain of the function X_j, i.e. $C_j = X_j$ (S).

This means that two sets are associated with each attribute A_j: the set of linguistic values

$${}_*A_j = \{ \lambda_{j1}, \lambda_{j2}, ..., \lambda_{ja(j)} \},$$

and the corresponding set of membership functions

$$G_j = \{\mu_{\lambda_{j1}}, \mu_{\lambda_{j2}}, ..., \mu_{\lambda_{ja(j)}}\} .$$

The n-dimensional Cartesian product of sets G_j:

$$G^n = \mathrm{G_1} \times G_2 \times ... \times \mathrm{G_n}$$

is referred to as the granuland of the structural parameters of social class.

This granuland forms the basis for the entire analysis of social class. An illustration of the granuland structure for a social class is shown in Fig. 1

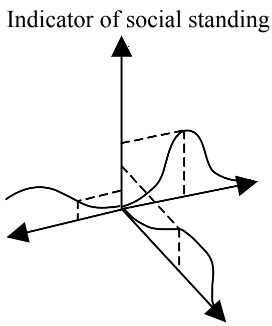

Fig. 1. Structure of a granuland

Within the framework of granulands, we introduce the basic concepts related to social class in the next section.

5. Conceptualization of social class

A social class is defined by expressions of the following kind:

$$\textit{if } A_1 \text{ is } \lambda^1, A_2 \text{ is } \lambda^2, ..., A_n \text{ is } \lambda^{\mathrm{n}}, \text{then Class is Sc},$$

where *Sc* is a linguistic value of the variable "Class",

and $\lambda^j \in {}_*A_j$.

Therefore, social class is identified by the vector

$$(\lambda^1, \lambda^2, ..., \lambda^n) \in {}_*\overline{A^n}.$$

The truth value of the sentence "A_j is λ^j" is interpreted as the "contribution" of attribute A_j in the determination of the class *sc*. This value is denoted by the symbol h_j. Formally, h_j is treated as a mapping

$$h_j : \mathrm{S} \to [0,1]$$

defined by the following composition:

$$h_j = \mu_{\lambda_j^*}(X_j(s)),$$

where λ_j^* is an appropriate element of the set ${}_*\overline{A_j}$, and s stands for a generic subject from the society S .

The value of this function calculated for a subject s_i is denoted as h_{ij}, i.e. $h_{ij} = \mu_{\lambda_j^*}(X_j(s_i))$.

In order to emphasise that the value h_j is obtained by only considering this particular aspect $\mathrm{A_j}$, the notation $h_j = \mathrm{h}(\mathrm{A_j})$ will be used.

Suppose that there is a society of scientists, which might be divided into classes such as: charlatans, researchers, mediocre ones , outstanding ones , geniuses etc.

As attributes for such a classification we can choose, for example, the number of publications ($\mathrm{A_1}$), number of citations ($\mathrm{A_2}$), and quality of publications ($\mathrm{A_3}$). Suppose that as linguistic values for the number of publications we choose the following: *small, negligible, fair, huge*. For the number of citations we choose such "values" as *insignificant, significant, enormous*. The quality of publications could be evaluated as *nebulous-verbosity, sorcery, pompous-bluff, creative, seminal*.

The class *outstanding scientists* can be defined as follows:

If the number of citations is *enormous,* the number of publications is *huge*, the quality of publications is *seminal*, then the Class is *outstanding scientists*.

A schematic presentation for the evaluation of this conditional assertion is shown in Fig. 2.

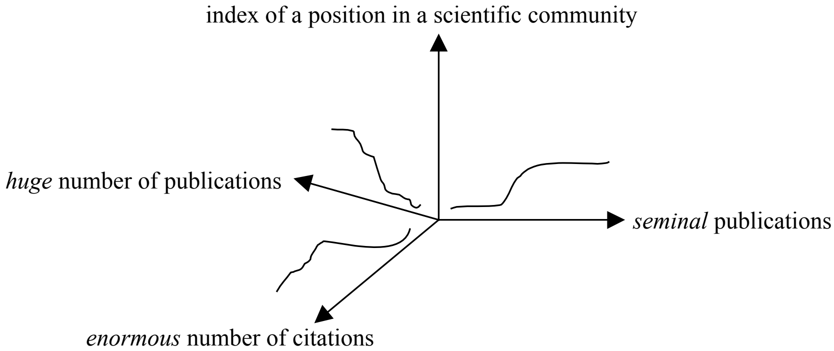

Fig. 2. Granulands determining the class of scientists

For example, the "contributions" of the three attributes selected for the classification of L. Zadeh as an *outstanding scientist* could be calculated as follows:

$$h_1 = \mu_{\text{enormous}}(number\_of\_citations(Zadeh)) = \mu_{\text{enormous}}(10{,}468) = 1\,,$$

$$h_2 = \mu_{\text{huge}}(number\_of\_publications(Zadeh)) = \mu_{\text{huge}}(?) = 1\,,$$

$$h_3 = \mu_{\text{seminal}}(quality\_of\_publications(Zadeh)) = \mu_{\text{seminal}}(100) = 1\,.$$

Note: the number of citations given in this example is the number compiled by R. Lou and is available on the BISC-Group website. For the calculation of the degree of membership to the set *seminal,* one can rank all works on a scale from 0 to 100 according to how seminal they are.

To evaluate the total "contribution" of all aspects in the determination of social class, we propose to use an appropriate aggregation operator or apply some fuzzy integral. Schematically:

$$h = integration\ of\ h_1, h_2, ..., h_n.$$

In cases where the indication of a subject is needed, then the following notation will be used:

$$h_{i\bullet} = integration\ of\ h_{i1}, h_{i2}, ..., h_{in}.$$

In order to integrate all the values h_j into one synthetic value, fuzzy integrals will be used. To define these integrals, suppose that the indexes of the attributes are permutated in such a way that

$$h_{(1)} = h(A_{(1)}) \leq h_{(2)} = h(A_{(2)}) \leq \ldots \leq h_{(n)} = h(A_{(n)}) .$$

Let the symbol $S_{(j)}$ denote the following set of attributes

$$S_{(j)} = \{A_{(j)}, A_{(j+1)}, \ldots, A_{(n)}\} .$$

Sugeno's integral can be, in this case, defined by the formula:

$$h = \max_{\mathrm{j}} \{\min(h_{(j)}, \mu(S_{(j)}))\} .$$

The alternative, the Choquet integral, has the following form:

$$h = \sum_{j=1}^{n} (h_{(i-1)} - h_{(j)}) \cdot \mu(S_{(j)})$$

assuming that $h_{(0)} = 0$.

5. Zadeh's standard application of granulands

The problem which we intend to solve now is the possibilistic evaluation of such sentences as "*subject s belongs to the class C*". We will confine ourselves to three classes: *lower, middle*, and *upper*; which will also be identified with the numerals 1, 2, and 3, respectively.

So, it is supposed that the society $S = \{s_1, s_2, \ldots, s_N\}$ is divided into these three classes acording to the attributes $A_1, A_2, \ldots, A_n$.

The contribution of the attribute A_j to subject s_i belonging to the class k is denoted by the symbol

$$h_{ij}^{k}, \quad i = 1, 2, \ldots, \mathrm{N}, j = 1, 2, \ldots, n, k = 1, 2, 3.$$

To denote the contribution of all the attributes $A_1, A_2, \ldots, A_n$, the symbol $h_{i\bullet}^{k}$ will be used. This means that all the values h_{i1}^{k}, h_{i2}^{k}, ..., h_{in}^{k} are integrated into one synthetic measure interpreted as the indicator of membership of the class k.

To evaluate whether or not an indicator is significant, we need the definition of the fuzzy sets: *lower*, *middle*, and *upper*. Let us observe that the indicator of social standing takes on values in the unit interval. Furthermore, observe that the fuzzy sets *lower*, *middle*, and *upper* are ordered sets. These observations suggest the use of the standard *Zadehian granuland*, defined on the unit interval, as the basis for the required evaluations.

From a formal point of view, this granuland is a particular subset of fuzzy fractions. From the substantive point of view, this granuland embeds the essential, intuitive information concerning the classification. This granuland, although not explicitly defined, has been used in almost all of Zadeh's work (see [22], [23], [25]).

Formally, the standard Zadehian granuland is defined as the set of all fuzzy subsets defined on the unit interval

$$\{\mu \mid \mu\colon [0, 1 \rightarrow [0, 1\}.$$

We assume that this family of fuzzy sets is partially ordered. Moreover, it is partitioned into three ordered classes μ_{low}, μ_{upper} and μ_{middle}, which are closed under all admissible operations.

It is assumed that the membership functions μ_{low}, μ_{upper} and μ_{middle} have the following properties:

1) μ_{low} is decreasing,
2) μ_{upper} is increasing,
3) μ_{middle} is symmetric with respect to the point 0.5 and is increasing in the interval [0, 0.5],
4) $\mu_{low}(0) = \mu_{middle}(0.5) = \mu_{upper}(1) = 1$,
5) $\mu_{low}(0.5) = \mu_{upper}(0.5)$.

Other properties can be defined depending on the area of applications. We can distinguish three separate situations.

First, one can require that each member of a society belongs (possibly with different degrees) to only one class. In such a situation the three fuzzy sets μ_{low}, μ_{upper} and μ_{middle} should be sharply mutually disjoint.

Second, if one permits each member of the society to belong to two adjacent classes, then it should be required that only adjacent fuzzy sets are overlapping.

In the third important case, a member of a society can belong simultaneously to all classes with different degrees of membership. In such a case, any two fuzzy sets overlap.

Schematically, these three situations are shown in Fig. 3

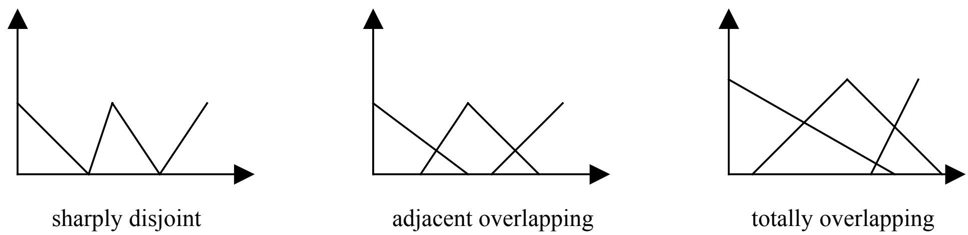

Fig. 3. Three kinds of fuzzy classification

Let us consider some examples.

The lower class can be defined, for example, by the following membership function

$\mu_{low}(x)$ = –2x + 1, for $0 \le x \le 0.5$, and 0 otherwise.

The middle class is defined by

$\mu_{middle}(x)$ = 2x, for $0 \le x \le 0.5$

$\mu_{middle}(x)$ = -2x +2 , for $0.5 \le x \le 1$.

For the upper class, the membership function has the form:

$\mu_{upper}(x)$ = 2x – 1 , for $0.5 \le x \le 1$.

If one wishes to divide the unit interval to satisfy the Bezdek condition for a fuzzy partition (see [10]), then the membership functions can be defined as follows:

$\mu_{low}(x)$ = x /(2x + 1), for $0 \le x \le 0.5$

$\mu_{middle}(x)$ = 2x/(2x + 1), for $0 \le x \le 0.5$

$\mu_{middle}(x)$ = (–2x +2)/(2x + 1), for $0.5 \le x \le 1$

$\mu_{upper}(x)$ = (1 – x)/(2x + 1), for $0.5 \le x \le 1$.

The size of a social class could be defined as the cardinality of the corresponding fuzzy subset, or as the cardinality of a supporting baseline subset defined by means of α-cuts.

6. Illustrative example

Let us consider a simple example. For the classification of a given society, the following three attributes are used:

Z – occupation (attribute A_1),

B – wealth (attribute A_2),

W – education (attribute A_3).

Some linguistic values are defined below:

$_*Z = \{\lambda_{11}, \lambda_{12}, \lambda_{13}, ...\} = \{prestigious, well-paid, ...\}$

$_*B = \{\lambda_{21}, \lambda_{22}, \lambda_{23}, ...\} = \{poor, rich, ...\}$

$_*W = \{\lambda_{31}, \lambda_{32}, \lambda_{33}, ...\} = \{elementaryschool, \sec ondaryschool, college...\}$

When only three attributes are considered, the three social classes *L*, *M*, and *U* are defined by the three triplets:

$$(\lambda_L^1, \lambda_L^2, \lambda_L^3), \quad (\lambda_M^1, \lambda_M^2, \lambda_M^3), \text{ and } \quad (\lambda_U^1, \lambda_U^2, \lambda_U^3).$$

Let us consider only one class, *U*, and suppose that it is defined by the triple

$$(\lambda_U^1, \lambda_U^2, \lambda_U^3) = (prestigious, not\_very\_poor, at\_least\_high\_school).$$

Assume that all three linguistic values are properly defined by the appropriate membership functions. These functions are not given here explicitly, but the hypothetical structure of these functions is illustrated in figure 4.

Consider two individuals, whose h_{ij} values are presented in table 1.

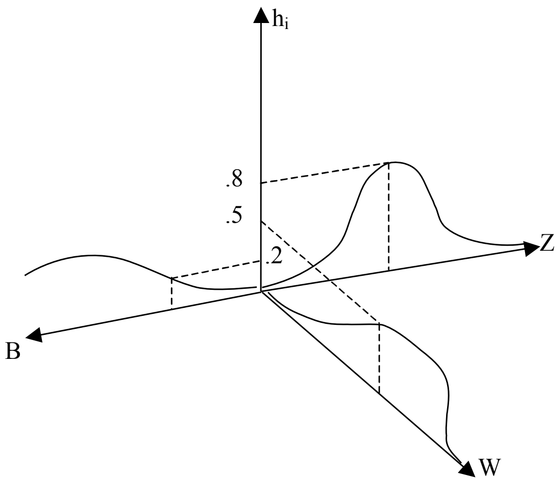

Fig. 4. Granuland of social structure

Table 1. Degrees of membership in classes Z, B, and W

Attributes / Members	Z	B	W
Individual 1	0.8	0.2	0.5
Individual 2	0.7	0.9	0.6
Etc.			

From this table (and also from the figure), we see that

$$h_1 = \mu_{\lambda_{11}}(occupation\_of\_1st\_individual)$$
$$= \mu_{prestigious}(occupation\_of\_1st\_individual) = 0.8$$

For the calculation of the value of the index of social position, we need to define a measure on the family of all subsets of the set {Z, B, W}. This measure is defined as follows:

$\mu(Z) = 0.4$

$\mu(B) = 0.3$

$\mu(W) = 0.4$

$\mu(\{Z,W\}) = 0.9$
$\mu(\{B,W\}) = 0.5$
$\mu(\{Z,B\}) = 0.8$
$\mu(\{Z,W\}) = 1.$

To evaluate the contribution of all three attributes to individual's 1 belonging to the upper class, we have to integrate the values $h_1 = 0.8$, $h_2 = 0.2$ and $h_3 = 0.5$ into one single value h. The Choquet integral will be used.

$$h = (0.2 - 0) \cdot 1 + (0.5 - 0.2) \cdot 0.9 + (0.8 - 0.5) \cdot 0.4$$
$$= 0.2 \cdot 1 + 0.3 \cdot 0.9 + 0.3 \cdot 0.4 = 0.2 + 0.27 + 0.12 = 0.41.$$

Then we have to calculate the degrees of membership to each of the three classes: *lower, middle*, and *upper*. Suppose that these classes are defined by the following fuzzy sets:

$\mu_{low}(x) = \exp(-\frac{9}{2}x^2)$,
$\mu_{middle}(x) = \exp(-18(x-0.5)^2)$,
$\mu_{upper}(x) = \exp(-4.5((x-1)^2)$.

The degrees of membership are then calculated as follows:

$\mu_{low}(h) = \exp(-\frac{9}{2}h^2) = \exp(-\frac{9}{2}\cdot 0.41^2) = 0.469$,
$\mu_{middle}(h) = \exp(-18(h-0.5)^2) = \exp(-18(0.41-0.5)^2) = 0.864$,
$\mu_{upper}(h) = \exp(-4.5((h-1)^2) = \exp(-4.5((0.41-1)^2) = 0.209$.

For individual 2, the indicator of social standing is calculated in the following way:

$$h = (0.6 - 0) \cdot 1 + (0.7 - 0.6) \cdot 0.8 + (0.9 - 0.7) \cdot 0.3$$
$$= 0.6 \cdot 1 + 0.1 \cdot 0.8 + 0.2 \cdot 0.3 = 0.6 + 0.08 + 0.06 = 0.74.$$

For this individual, we have

$\mu_{low}(h) = \exp(-\frac{9}{2}h^2) = \exp(-\frac{9}{2}\cdot 0.74^2) = 0.085$
$\mu_{middle}(h) = \exp(-18(h-0.5)^2) = \exp(-18(0.74-0.5)^2) = 0.354$
$\mu_{upper}(h) = \exp(-4.5((h-1)^2) = \exp(-4.5((0.74-1)^2) = 0.737$.

Comparing these degrees, we can assert that indivudual 1 is rather middle class, while individual 2 is definitely a member of the upper class.

For illustrative purposes, we will now calculate the indicators of social standing using the Sugeno integral.

For individual 1, we have:

$$h = \max_i \{\min(x_{(i)}, \mu(A_{(i)}))\}$$

$$= \max\{\min(0.2, \mu(B); \min(0.5, \mu(\{W,Z\})); \min(0.8, \mu(\{Z\}))\}$$

$$= \max\{0.2; 0.5; 0.4\}= 0.5.$$

And for individual 2:

$$h = \max\{\min(0.6;1); \min(0.7;0.8), \min(0.9; 0.3)\} = \max\{0.6; 0.7; 0.3\} = 0.7$$

The degrees of membership to the social classes can be determined as in the case where the Choquet integral was used.

7. Conclusions

Let us summarize the main steps of the procedure determining the degrees of membership of all members of the society investigated to the given social classes.

Each subject from the society S is identified with a vector

$$x^i = (x_{i1}, x_{i2}, ..., x_{in}) .$$

Table 2. Summary of the procedure for evaluating social class

Subject	Class	A_1	A_2	...	A_n	integrated contribution	degree of membership
s_1	1 = *low* 2 = *middle* 3 = *upper*	h_{11}^1 h_{11}^2 h_{11}^3	h_{12}^1 h_{12}^2 h_{12}^3		h_{1n}^1 h_{1n}^2 h_{1n}^3	$h_{1\cdot}^1$ $h_{1\cdot}^2$ $h_{1\cdot}^3$	$\mu_{low}(h_{1\cdot}^1)$ $\mu_{middle}(h_{1\cdot}^2)$ $\mu_{upper}(h_{1\cdot}^3)$
s_2	1 = *low* 2 = *middle* 3 = *upper*	h_{21}^1 h_{21}^2 h_{21}^3	h_{22}^1 h_{22}^2 h_{22}^3		h_{2n}^1 h_{2n}^2 h_{2n}^3	$h_{2\cdot}^1$ $h_{2\cdot}^2$ $h_{2\cdot}^3$	$\mu_{low}(h_{2\cdot}^1)$ $\mu_{middle}(h_{2\cdot}^2)$ $\mu_{upper}(h_{2\cdot}^3)$
$\vdots$							
s_N	1 = *low* 2 = *middle* 3 = *upper*	h_{N1}^1 h_{N1}^2 h_{N1}^3	h_{N2}^1 h_{N2}^2 h_{N2}^3		h_{Nn}^1 h_{Nn}^2 h_{Nn}^3	$h_{N\cdot}^1$ $h_{N\cdot}^2$ $h_{N\cdot}^3$	$\mu_{low}(h_{N\cdot}^1)$ $\mu_{middle}(h_{N\cdot}^2)$ $\mu_{high}(h_{N\cdot}^3)$

Such vectors are obtained by some scaling or measurement procedure. These procedures are not disscused in this paper. Then, vector $x^i = (x_{i1}, x_{i2}, ..., x_{in})$ is used to calculate the vector (h_{i1}^k, h_{i2}^k, ..., h_{in}^k) of contribution of each attribute to subject's s_i belonging the to the class k. By integrating all values h_{i1}^k, h_{i2}^k, ..., h_{in}^k into one single value $h_{i\bullet}^k$, we obtain the index of social standing. To evaluate this standing, the degrees of membership to all classes are to be determined. In table 2 the entire procedure is presented in a form adapted to computer implementaion.

References

[1] BATYRSHIN I., *On linguistic representation of quantitative dependences*, Expert Systems with applications, 2004, 26, 95–104.

[2] BLACK T.R., *Doing quantitative Research in the social sciences*, Sage Publications, 1999.

[3] BLAU P.M., *Inequality and heterogeneity, a primitive theory of social structure*, The Free Press, New York 1977.

[4] BLAU P.M., DUNCAN O.D., *The American occupational structure*, Wiley, New York 1967.

[5] DUBOIS D., PRADE H., *Fuzzy sets and systems: theory and applications*, New York, 1980.

[6] ESPING-ANDERSEN G., *Social foundations of post industrial economics*, Oxford University Press, new York 1999.

[7] GUTTMAN L., *A revision of Chapin's social status scale*, American Sociological Review, 1943, No. 3, 362–369.

[8] HATT P.K., *Occupation and social stratification*, American Journal of Sociology, 1950, 6, 533–543.

[9] HOUT M., *Otis Dudley Duncan's major contributions to the study of social stratification*, Research in Social Stratification and Mobility, 2007, 25, 109–118.

[10] KLIR G.J., FOLGER T.A., *Fuzzy sets*, Englewood Cliffs, 1988.

[11] KOHN M.L., SŁOMCZYŃSKI K.M., SCHOENBACH C., *Social stratification and the transmission of values in the family: a cross-national assessment*, Sociological Forum, 1986, No. 1, 73–102.

[12] KUNDU S., *The normal form of a granular fuzzy function*, Fuzzy Sets and Systems, 2001, 124, 97–107.

[13] PEDRYCZ W., KWAK K.C., *Boosting of granular models*, Fuzzy Sets and Systems, 2006, 157, 2934–2953.

[14] PIZZI N.J., PEDRYCZ W., *Effective classification using feature selection and fuzzy integration*, Fuzzy Sets and Systems, 159, 2008, 2859–2872.

[15] RAGIN CH.C., PENNINGS P., *Fuzzy sets and social research*, Sociological Methods and Research, 2005, 4, 423–430.

[16] SCHOOLER C., SCHOENBACH C., *Social class, occupational status, occupational self-direction, and job income. A cross-national examination*, Sociological Forum, 1994, Vol. 9, 431–457.

[17] SMITH T., GRAHAM P., *Socioeconomic stratification in family research*, Journal of Marriage and Family, 1995, 57, 930–940.

[18] SMITH T.E., GRAHAM P.B., *Socioeconomic stratification in family research*, Journal of Marriage and the Family, 57, 1995, 930-940.

[19] SZTOMPKA P., *System and Function. Towards a theory of Society*, Academic Press, New York 1974.

[20] VERKUILEN J., *Assigning membership in a fuzzy set analysis*, Sociological Methods and Research, 2005, Vol. 33, 462–496.

[21] WEBER S., *⊥-Decomposable Measures and Integrals for Archimedean t-conorms ⊥*, Journal of Mathematical Analysis and Applications, 1984, No. 1, 114–138.

[22] ZADEH L., *Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic*, Fuzzy Sets and Systems, 1997, 90, 111–127.
[23] ZADEH L., *Fuzzy Sets and information granularity*, [in:] *Advances in Fuzzy Set Theory and Application*, M. Gupta, R. Ragade and R. Yager (eds.), North-Holland Publishing Co., Amsterdam 1979, 3–18.
[24] ZADEH L., *Fuzzy logic*, [in:] *Encyclopedia of complexity and system science*, Mayers (ed.), 2008.
[25] ZADEH L.A., *A computational approach to fuzzy quantifiers in natural languages*, Computers and Mathematics, 1983, Vol. 9, 149–184.
[26] ZIMMERMANN H.-J., *Fuzzy sets theory*, Kluwer, 1991.