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Letters to the Editor

Astigmatism and field curvature of hybrid imaging surface

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Hybrid imaging surface is an optical surface separating two media of different refraction indices with a diffraction structure deposited on it. We assume that diffraction structure has geometry identical with an interference pattern generated by interference of two spherical waves. For such a surface we found formulae expressing location of meridional and sagittal foci of astigmatic ray pencil, analogous to *s*- and *t*-tracing formulae known in the case of the purely refractive surface. We used the said formulae for correction of astigmatism/field curvature of hybrid lens by appropriate shift of input pupil.

1. Introduction

Both phenomena, classic light refraction as well as diffraction on periodic or quasi -periodic structure deposited on an optical surface separating two optical media can be used for obtaining optical imaging. Diffractive-refractive (hybrid) optical imaging elements and systems are nowadays used practically in all possible applications [1]. Due to the rapid development of manufacturing technology it is possible to generate the desired diffraction pattern either interferometrically or synthetically (*e.g.*, by microprinting process) almost without limitations.

In order to evaluate the potential imaging quality of such lens it is necessary to study its aberration characteristics. The most frequent description of image quality includes the Seidel aberrrations [2]–[4]. Such an approach, based on the III-order approximation, may not be sufficient, especially for greater field angles. Therefore, it is worth trying to develop the formulae describing the parameters of light rays in astigmatic pencil enabling us to investigate astigmatism and field curvature without restriction following from the III-order approximation.

2. Preliminaries

2.1. Refraction on diffractive surface

Let us assume that plane surface with a diffractive surface deposited on it separates two media of different indices of refraction n and n'. The monochromatic (of wavelength λ_0) light pencil falling onto this surface at the angle i in the first medium changes its direction in the second medium due to refraction and diffraction.

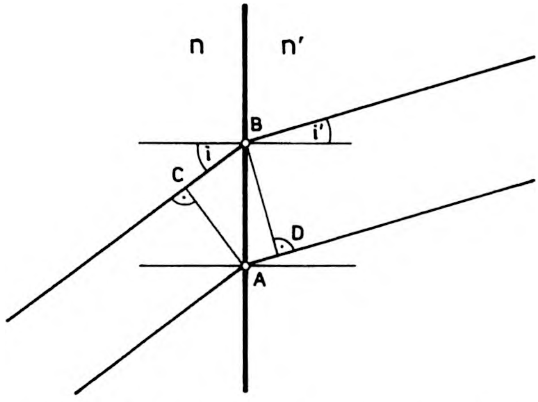

Fig. 1. Deviation of light rays on hybrid surface.

The new direction of the ray is described by the angle i'. The relationship between both angles can be found from the equation

$$\overline{AD}n' - \overline{CB}n = \pm\lambda_0, \tag{1}$$

which follows immediately from Fig. 1 and condition of constructive interference [5]. Assuming that $\overline{AB} = D$ is the fringe spacing we have

$$n'\sin i' - n\sin i = \pm\frac{\lambda_0}{D}. \tag{2}$$

This equation expresses the generalised Snells law describing light refraction on diffractive surface.

2.2. Fringed structure of diffractive lens

We consider here the diffractive lens specified by fringe geometry analogous to that obtainable by the interference of two spherical waves originating from the points P_α and P_β located on the lens axis at the distance z_α and z_β from the lens, respectively (see Fig. 2).

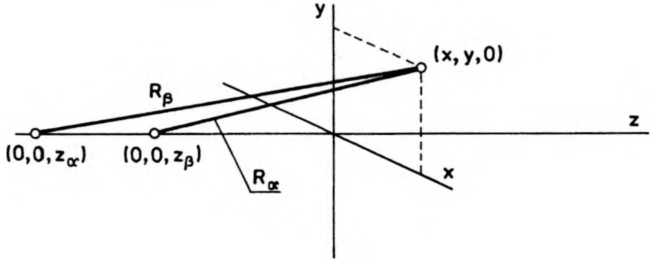

Fig. 2. Generation of hybrid surface.

Fringe spacing D at an arbitrary point $P=(x,y,0)$ on the lens surface can be found from the condition

$$D\,\mathrm{grad}(\Delta\Phi)=2\pi \tag{3}$$

where $\Delta\Phi$ is the phase difference between both spherical waves.

Since

$$\Delta\Phi=\frac{2\pi}{\lambda_0}(R_\alpha-R_\beta) \tag{4}$$

where

$$R_\alpha=\mathrm{sgn}\,(z_\alpha)\sqrt{x^2+y^2+z_\alpha^2}, \tag{5a}$$

$$R_\beta=\mathrm{sgn}(z_\beta)\sqrt{x^2+y^2+z_\beta^2}, \tag{5b}$$

we have vertical and horizontal components of fringe spacing as

$$D_x=\frac{\lambda_0}{\varphi_D x}, \tag{6a}$$

$$D_y=\frac{\lambda_0}{\varphi_D y} \tag{6b}$$

where

$$\varphi_D=\frac{1}{R_\alpha}-\frac{1}{R_\beta} \tag{7}$$

is local focusing power of the diffractive structure.

3. The s- and t-tracking formulae for meridional and sagittal foci

3.1. Meridional cross-section

Let us consider a meridional cross-section of plane diffractive surface with input pupil shifted in front of it at the distance d as presented in Fig. 3. The chief ray connecting the object point P and the input lens centre intersects the lens plane at the point A located off axis and here it deviates towards the image point P'. The other ray, infinitesimally close to the first one, intersects the lens plane at the point B. The angles of incidence and deviation of both rays are marked in the picture.

We can calculate the segments t_m and t'_m from the lens plane to the object and image points respectively in the following way. By differentiating Eq. (2) we have

$$n'\cos i'\,di'=n\cos i\,di\pm\frac{\lambda_0}{D_y^2}\,\mathrm{d}D_y \tag{8}$$

where derivative of D_y (given by Eq. (6b)) is as follows:

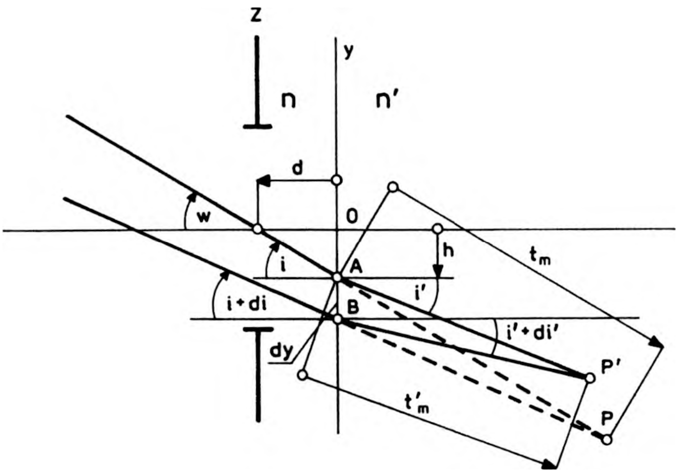

Fig. 3. Astigmatic light pencil in meridional cross-section.

$$dD_y = -\left[\frac{\lambda_0}{\varphi_D y^2} - \frac{\lambda_0}{\varphi_D^2}\left(\frac{1}{R_\alpha^3} - \frac{1}{R_\beta^3}\right)\right] dy = -\frac{\lambda_0}{\varphi_D^2 y^2}\left[\varphi_D - y^2\left(\frac{1}{R_\alpha^3} - \frac{1}{R_\beta^3}\right)\right] dy, \tag{9}$$

and thus

$$\frac{\lambda_0}{D_y^2} dD_y = \mu\left[\varphi_D - y^2\left(\frac{1}{R_\alpha^3} - \frac{1}{R_\beta^3}\right)\right]. \tag{10}$$

From the triangles ΔABP and $\Delta ABP'$ we have:

$$\frac{AB}{\sin(di)} = \frac{t_m}{\sin\left(\frac{\pi}{2} + i + di\right)}, \tag{11a}$$

$$\frac{AB}{\sin(di')} = \frac{t'_m}{\sin\left(\frac{\pi}{2} + i' + di'\right)}. \tag{11b}$$

The segment $AB = dy$ is infinitesimally small, so are the angles di and di', therefore

$$di = \frac{\cos i}{t_m} dy. \tag{12a}$$

$$di' = \frac{\cos i'}{t'_m} dy. \tag{12b}$$

After substituting Eqs. (10), (13a) and (13b) into (8) and straightforward calculations we have

$$n' \frac{\cos^2 t'}{t'_m} = n \frac{\cos^2 i}{t_m} \pm \left[\varphi_D - y^2\left(\frac{1}{R_\alpha^3} - \frac{1}{R_\beta^3}\right)\right]. \tag{13}$$

Let us note that due to shift of the input pupil with respect to the lens plane the point A where chief ray intersects the lens lies outside the lens axis. Since the diffractive fringe spacing depends linearly on the height behind the axis the last term on the right hand side of Eq. (13) appears.

3.2. Sagittal cross-section

The sagittal cross-section of the lens considered is presented in Fig. 4 [6]. The chief ray and its close neighbour intersect the lens plane at the points A and B separated by the segment $\overline{AB} = \mathrm{d}x$. By proceeding in a similar way as in the previous paragraph we can write

$$n'\cos i'\, di' = n\cos i\, di \pm \frac{\lambda_0}{D_x^2}\,\mathrm{d}D_x. \tag{14}$$

In the equation analogous to (10) we have $x = 0$ and therefore the last term on the right hand side of the equation should be omitted

$$\frac{\lambda_0}{D_x^2}\,\mathrm{d}D_x = \mu\varphi_D \mathrm{d}x. \tag{15}$$

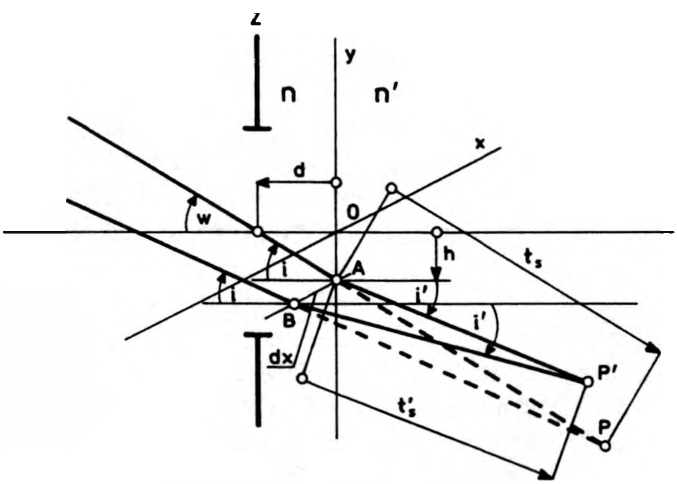

Fig. 4. Astigmatic light pencil in sagittal cross-section.

From the triangles ΔOAP and $\Delta OAP'$ we have:

$$\frac{\overline{OA}}{\sin(di)} = \frac{t_s \cos i}{\sin(\pi/2 - di)}, \tag{16a}$$

$$\frac{\overline{OA}}{\sin(di')} = \frac{t'_s \cos i'}{\sin(\pi 2 - di)}. \tag{16b}$$

The segment $\overline{OA} = \mathrm{d}x$ is infinitesimally small, so are the angles di and di', therefore

$$di = \frac{\mathrm{d}x}{t_m \cos i}, \tag{17a}$$

$$di' = \frac{\mathrm{d}x}{t'_m \cos i'}. \tag{17b}$$

By composing (14), (15), (17a) and (17b) we receive

$$\frac{n'}{t'_s} = \frac{n}{t_s} \pm \varphi_D. \tag{18}$$

The formulae (13) and (18) are fully corresponding to the so-called "s- and t-tracing formulae by Young" [7]. If we assume lack of diffraction our formulae transform into the known equations [7]–[9].

It is worth noting that for classic refractive surface the formulae discussed do not depend formally on the location of input pupil. Such dependence exists, of course, but it is not straightforward. It is the value of angle i which depends on the field angle ω and the value of pupil shift d. For hybrid surface, however, in formula (13) for meridional focus, there exists a term which is related directly to the coordinate y of the point at which the chief ray intersects the hybrid surface. For a given field angle ω the value of y is directly proportional to the input pupil shift (see Figs. 3 and 4).

The formulae derived are useful for calculation of the position of the meridional and sagittal images in an optical system composed of refractive and flat hybrid surfaces. Using them we can evaluate astigmatism or field curvature of hybrid lens. It is also possible to find the location of input pupil which assures correction of these aberrations.

4. Numerical example and conclusions

In order to illustrate the usefulness of the derived formulae for correction of astigmatism and field curvature we considered the simplest possible hybrid lens composed of a flat glass substrate of refraction index n and thickness t and a diffractive structure generated on its first surface. The fringed structure corresponds to the interference pattern of two spherical waves of centres located at points distant by z_α and z_β from the first surface. Such a structure need not be generated by interference of real waves (*i.e.*, by holographic method), but most often it is manufactured synthetically with the help of one of many available microprinting devices.

We considered an object located at infinity (focusing lens). Using formulae (13) and (18) and typical procedure as in the lens design and aberration correction process [7]–[9] we found the locations of input pupil assuring correction of astigmatism d_A and field curvature d_F for a given maximum field angle.

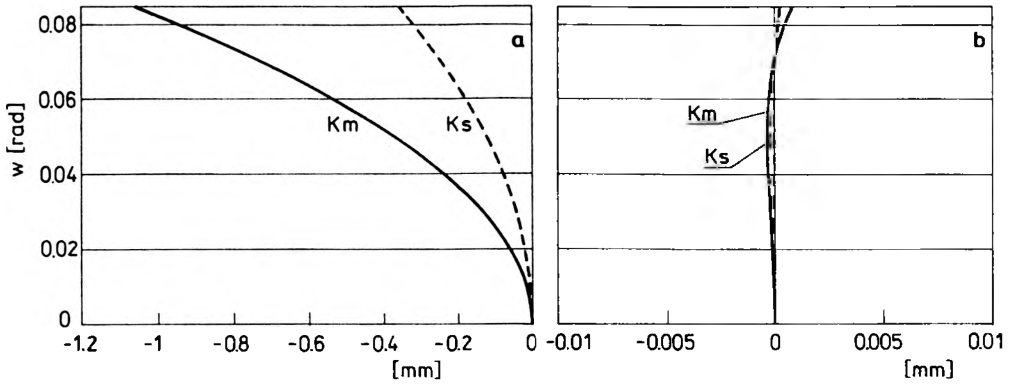

Fig. 5. Astigmatism and field curvature of the lens considered with input pupil: **a** – in contact, **b** – shifted by $d = -4.9$ mm.

The numerical values of the lens and imaging parameters are given in the Table. The results of aberration correction are illustrated in Fig. 5. Curves in Fig. 5**a** correspond to the lens with input pupil in contact. Astigmatism and field curvature are not corrected. In Figure 5**b** both aberrations are compensated almost completely by shifting the input pupil by the d_{AF} being a compromise between the values d_A and d_F. As one can see from the picture both aberrations are very well corrected.

Table. Parameters of exemple hybrid lens and imaging conditions.

$r_1 = r_2$	n	t	z_α	z_β	s	d_A	d_F	d_{AF}
∞	1.5	10 mm	100 mm	$-\infty$	$-\infty$	-44.0 mm	-54.4 mm	-49.09mm

The formulae for meridional and sagittal foci are given here only for flat surfaces and are not applicable if the diffractive structure is deposited on non-plane (*e.g.*, spherical) surface. In the latter case the analytic form of analogous formulae became too complicated to be useful in practice. It seems that in the days of fast and efficient computers numerical methods can substitute the analytical calculations in such a situation.

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