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Image contrast in aberrated coherent optical systems with cosinusoidal filters in their pupils

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In the paper, a change in contrast of both amplitude and phase tests in aberrated coherent optical systems equipped with cosinusoidal apodisation in the pupil is shown.

1. Introduction

In papers [1]–[3], it has been shown that the introduction of an amplitude-phase apodiser in an aberrated coherent optical system results in the respective change of contrast, when imaging a periodic amplitude or phase object. This change depends on the modulation depth of the test and on the amplitude part of the function describing the apodisation filter. The change of contrast was examined in [1]–[2] as related to the apodisation filter as well as to the aberrations of the optical system equipped with apodisers of type $[1/2(1+r^2)]^p$, $(1-r^2)^p$, $(1-|r|)^p$, for $p = 1, 2, 3, 4$, while in [3] for the apodiser of Hamming filter $0.54+0.46\cos(2\pi r)$, Gaussian filter $\exp(-r^2)$, Riecz filter $1-(r/2)^2$, and Lanczo's filter $\sin(2\pi r)/(2\pi r)$ types. In this paper, a change in the image contrast of an amplitude test of modulation depth $m = 0, 0.5, 1.0$ as influenced by the amplitude apodisation defined by $t(r) = \cos(Nr)$ and $t(r) = \cos^2(Nr)$ for $N = 1, 3, 5, 9$, and a change in the image contrast for an amplitude and phase test of modulation depth $m = 0$ in a coherent optical system with aberrations $w(r) = 0.5\lambda r^2, \lambda r^2, 2\lambda r^2$ apodised with filters $\cos(Nr)$ and $\cos^2(Nr)$ for $N = 1, 3, 5$, are examined.

2. Basic relations

The amplitude-phase transmittance of an apodiser in the exit pupil of a coherent optical system has the form

$$A(r) = t(r)e^{i\Phi(r)}, \quad 0 \le r \ge 1. \tag{1}$$

If the wave aberration $w(x, y)$ in taken into account in an optical system of rotational symmetry the total change of phase in the pupil is

$$W(x, y) = w(x, y)+\Phi(r), \quad r = \sqrt{x^2+y^2}. \tag{2}$$

In this paper the influence of the amplitude apodisers (for which $\Phi(r) = 0$ is assumed) on the change of contrast in the image of the periodic amplitude and phase tests is examined.

2.1. Amplitude test

If in the object space of an optical system a test of amplitude transmittance is inserted

$$H(x, y) = a + b\cos(2\pi f_x x) \tag{3}$$

the Michelson contrast of this object test becomes

$$K = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} = \frac{2ab}{a^2 + b^2}. \tag{4}$$

As shown in papers [1], [2] the contrast in the image of such a test is equal to

$$K'(f_x) = \frac{2abt(0)t(s)}{a^2t^2(0) + b^2t^2(s)}\cos k\left[\frac{W(s) + W(-s)}{2} - W(0)\right], \tag{5}$$

$$s = \frac{\lambda f_x R}{f_g}, \quad k = \frac{2\pi}{\lambda}$$

where f_x – spatial frequency, f_g – cut-off frequency, R – reference sphere radius, λ – wavelength.

From the comparison of (4) and (5) the change in contrast in the image as related to the object contrast (see [1], [2]) can be estimated as

$$D(f_x) = \frac{K'(f_x)}{K(f_x)} = \frac{\frac{t(s)}{t(0)}(1 + m^2)}{1 + m^2\frac{t^2(s)}{t^2(0)}}\cos k\left[\frac{W(s) + W(-s)}{2} - W(0)\right] \tag{6}$$

where $m = b/a$ – depth of test modulation.

The change of phase in the image is equal to $\theta(f_x) = k(W(s) + W(-s))/2$. From formula (5) it follows that the change in phase is independent of the apodiser introduced to the optical system.

The introduction of the apodiser results in a change of contrast depending on the modulation depth. For a low-contrast object $(m \to 0)$, when $t(0) \to 0$, the contrast in the image improves significantly. This technique is known in optics as amplitude contrast. For a high-contrast object $(m \to 1)$ the apodiser lowers the contrast in the image.

2.2. Phase test

For the phase test of transmittance

$$H(x,y) \approx 1 + im\sin(x), \tag{7}$$

the change in image contrast as related to the object contrast is of the form [1], [2]

$$D(f_x) = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}\sin k\left[\frac{W(s)+W(-s)}{2} - W(0)\right]. \tag{8}$$

From the Eq. (8) it follows that for low-contrast objects, *i.e.*, for $m \to 0$ when $W(0) = \pi/2$ and $W(s) = 0$, the contrast in the image is greatly improved (this technique is known as phase contrast). In the function describing the change in contrast of the amplitude (6) and phase (8) tests, a part depending only on the respective part of the apodizing function $t(s)$ and a part depending on the aberration $w(x,y)$ of the optical system can be distinguished. If D_t denotes the first part only, then

$$D_t = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}. \tag{9}$$

For the test of small modulation depth $(m \to 0)$ we obtain

$$D_{t,\, m\to 0} = \frac{t(s)}{t(0)}.$$

3. Results and discussion

The function $D_t(s)$ for apodisers of types: $\cos(Nr)$ and $\cos^2(Nr)$ is shown in Figs. 1**a**–**g** and 2**a**–**f**. The apodisers of this type lower the contrast for $N = 1$, while the contrast decrease is the less the less the modulation depth. Additionally, for the high frequencies the contrast reversal is observed (Fig. 1**b**) for $N = 3$. For $N = 5$ (Fig. 1**e**) the contrast reversal is observed for medium and high frequencies. For $N = 9$ (Fig. 1**d**) the contrast reversal is observed twice for all the modulation depth in the case of filter $\cos(Nr)$. The contrast does not practically differ for high and small modulation depths (Fig. 1**e**–**g**). In the case of the filter $\cos^2(Nr)$ high contrast for great modulation depths and low contrast for small modulation depths are observed for $N = 1$ (Fig. 2**a**). For $N = 3$ (Fig. 2**b**) the contrast drops down to zero for medium frequencies and next increases. We have a similar situation for $N = 5$ (Fig. 2**c**). The contrast is practically the same for all the modulation depths

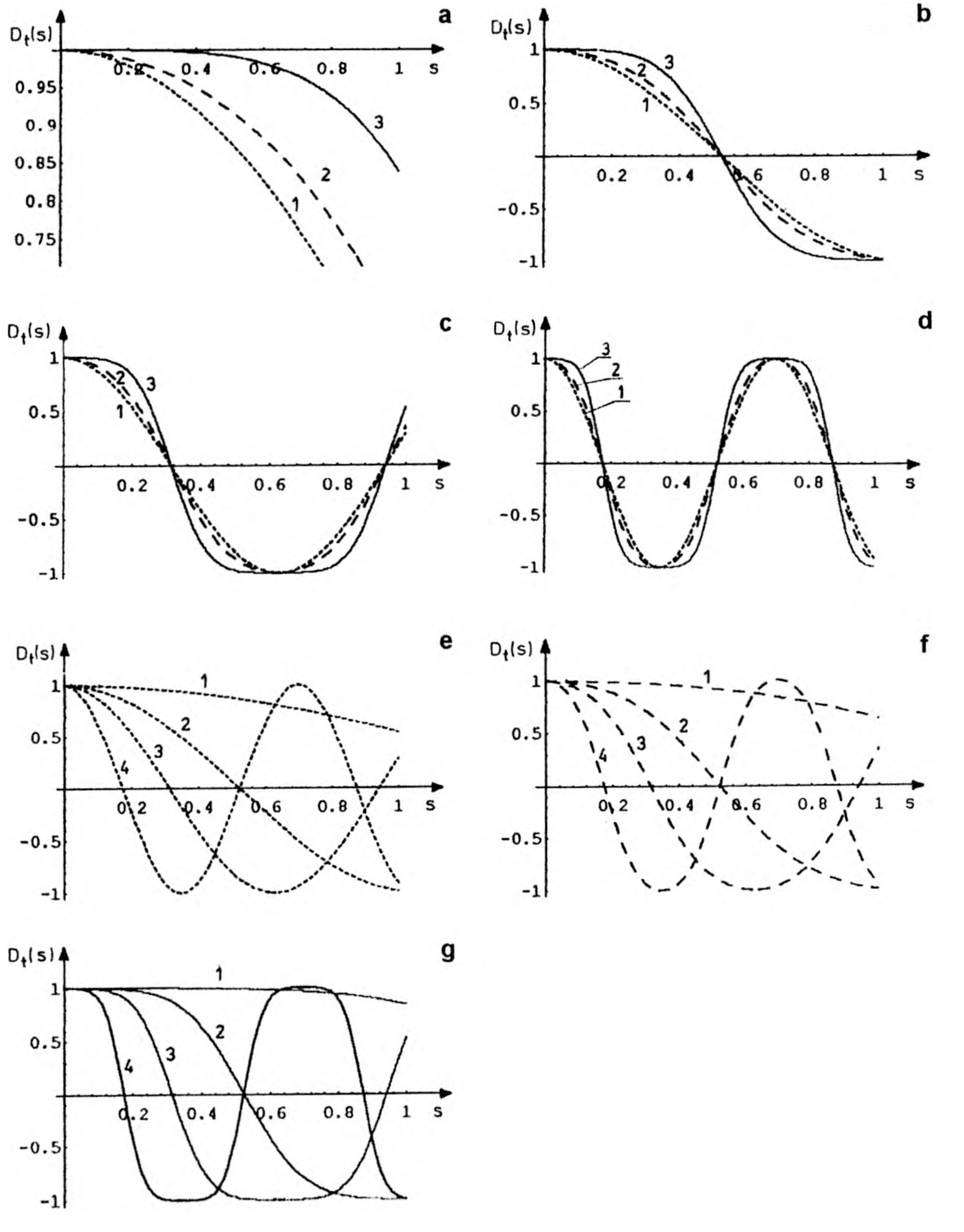

Fig. 1. Effect of amplitude apodisation $t(r) = \cos(Nr)$ on the contrast of image of the amplitude object test for: **a** – $N = 1$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **b** – $N = 3$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **c** – $N = 5$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **d** – $N = 9$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **e** – $m = 0$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3), $N = 9$ (4), **f** – $m = 0.5$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3), $N = 9$ (4), **g** – $m = 1.0$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3), $N = 9$ (4).

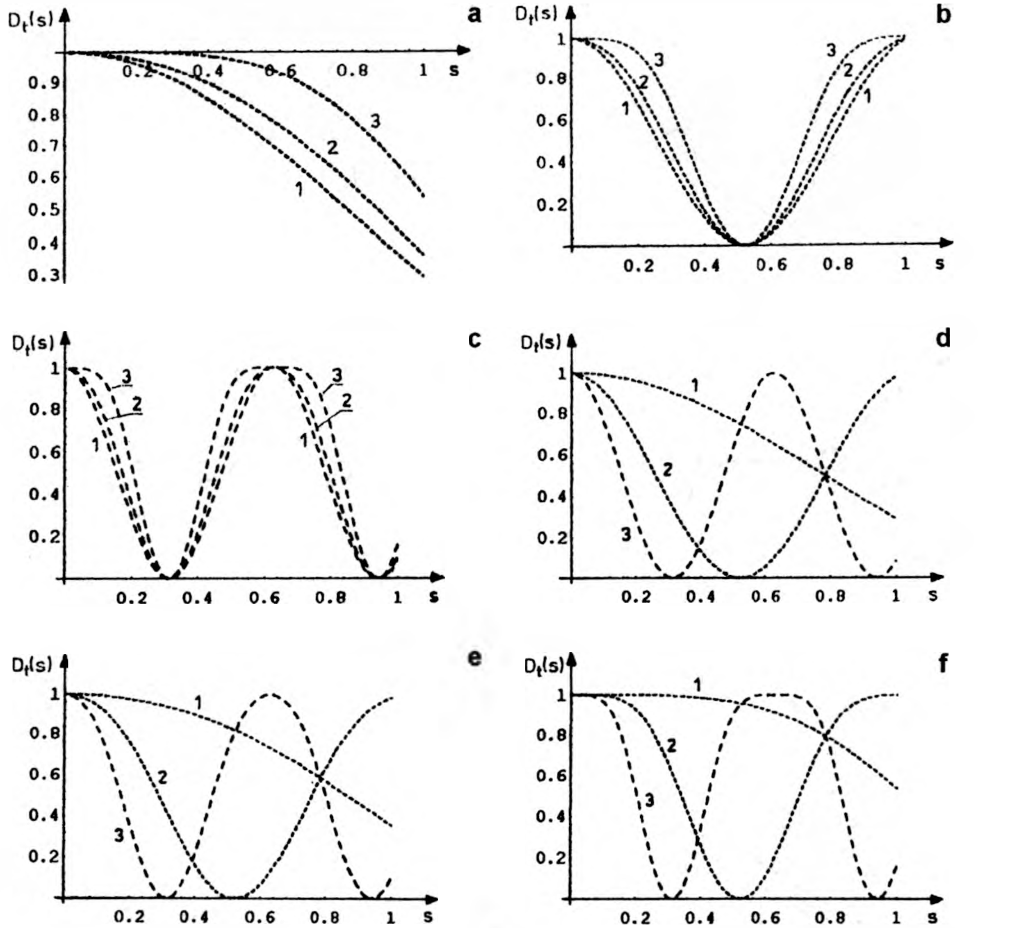

Fig. 2. Effect of amplitude apodisation $t(r) = \cos^2(Nr)$ on the contrast of image of the amplitude object test for: **a** – $N = 1$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **b** – $N = 3$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **c** – $N = 5$, $m = 0$ (1), $m = 0.5$ (2), $m = 1.0$ (3), **d** – $m = 0$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3), **e** – $m = 0.5$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3), **f** – $m = 1.0$, $N = 1$ (1), $N = 3$ (2), $N = 5$ (3).

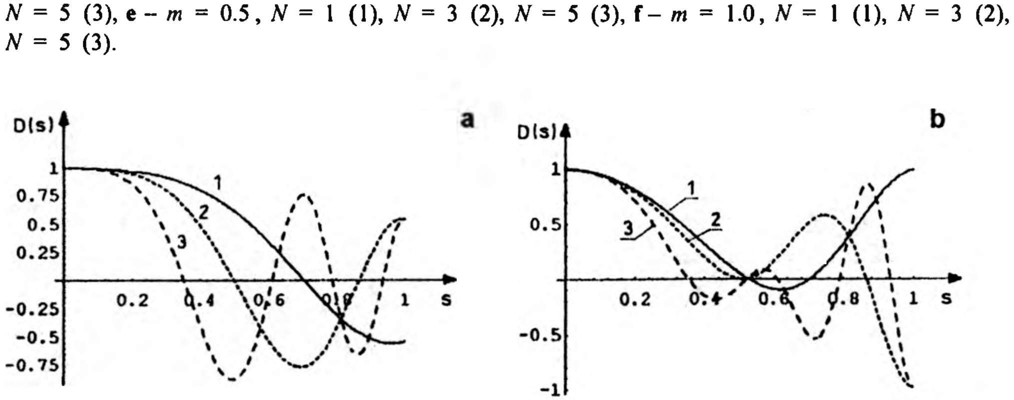

Fig. 3**a**, **b**

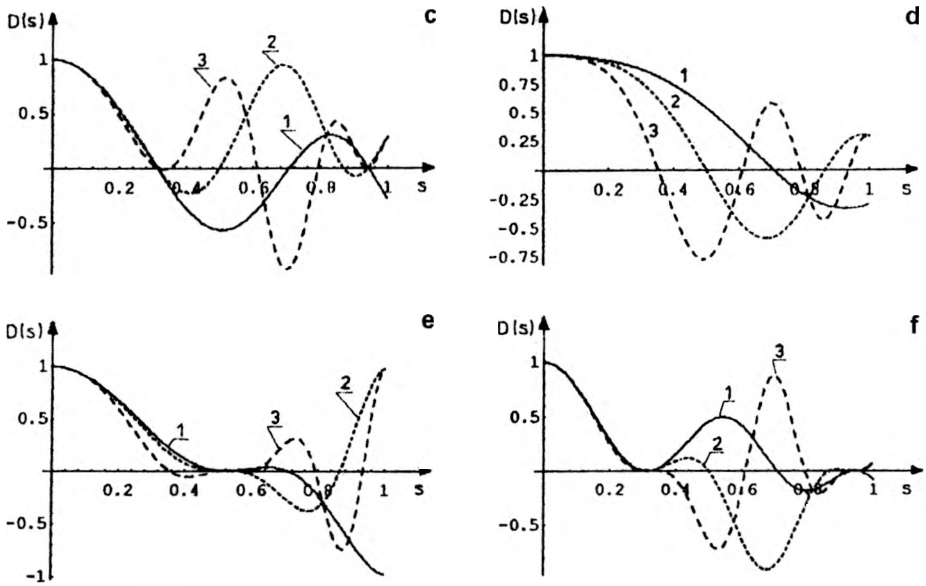

Fig. 3. Change in contrast $D(s)$ for the amplitude test of the modulation depth $m = 0$ in the optical system with aberrations $w(r) = 0.5\lambda r^2$ (1), $w(r) = \lambda r^2$ (2), $w(r) = 2\lambda r^2$ (3) apodised with filters $\cos(Nr)$ for: **a** – $N = 1$, **b** – $N = 3$, **c** – $N = 5$, and apodised with filters $\cos^2(Nr)$ for: **d** – $N = 1$, **e** – $N = 3$, **f** – $N = 5$.

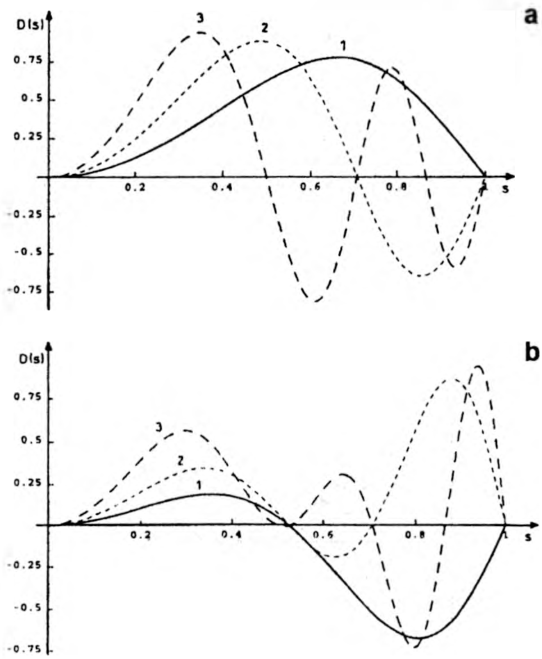

Fig. 4**a**, **b**

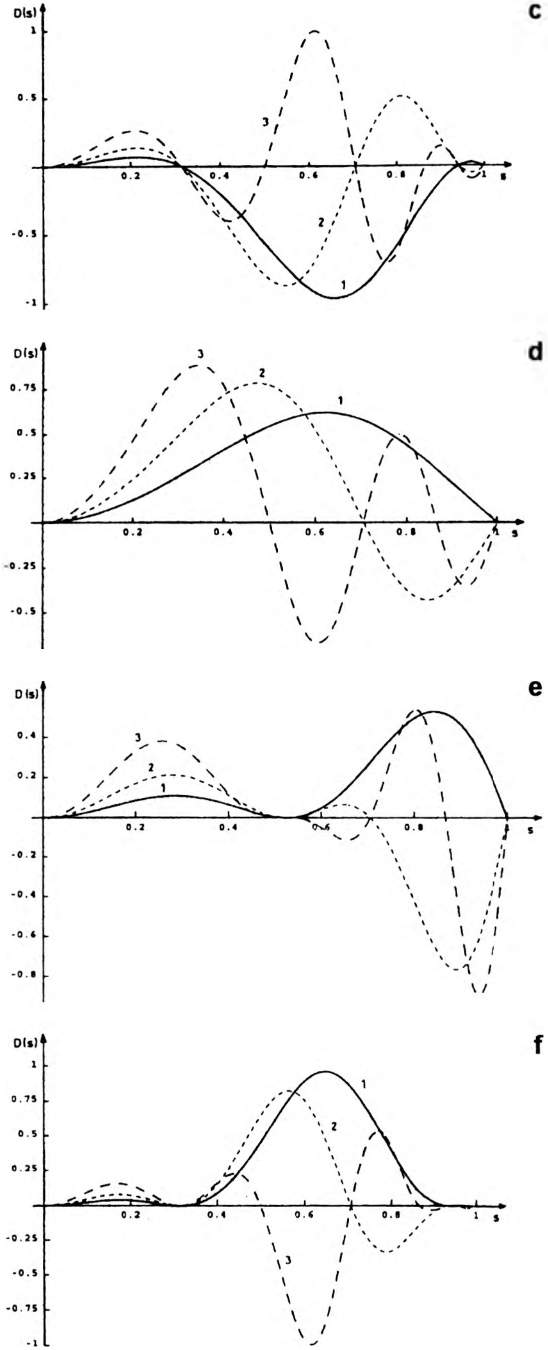

Fig. 4. Change in contrast $D(s)$ for the phase test of the modulation depth $m = 0$ in the optical system with aberrations $w(r) = 0.5\lambda r^2$ (1), $w(r) = \lambda r^2$ (2), $w(r) = 2\lambda r^2$ (3) apodised with filters $\cos(Nr)$ for: **a** – $N = 1$, **b** – $N = 3$, **c** – $N = 5$ and apodised with filters $\cos^2(Nr)$ for: **d** – $N = 1$, **e** – $N = 3$, **f** – $N = 5$.

$m = 0, 0.5, 1.0$ (Fig. 2**d**–**f**). The change of contrast $D(s)$ in the optical system apodised by the filters $\cos(Nr)$ and $\cos^2(Nr)$ for an amplitude and phase object tests of small modulation depths $m = 0$ for $N = 1, 3, 5$ and aberration $w(r) = 0.5\lambda r^2$, λr^2, $2\lambda r^2$ is shown in Figs. 3**a**–**f** and 4**a**–**f**, respectively. As it follows from the figures the introduction of the aberrations worsens the contrast in the case of amplitude object test (Fig. 3) but improves contrast of the phase object test (Fig. 4). The contrast is maximal for non-aberrated optical system and it becomes highly diminished with the increased aberrations for medium and high frequencies.

References

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