Optica Applicata, Vol. XXXII, No. 4, 2002

New shape of a videokeratometric illuminator

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The aim of this paper is to define such shape of a videokeratometric Placido disc-based illuminator that would guarantee flat image formation and, consequently, sharp and reliable information an corneal geometry. It is especially important for contact lens fitting to have reliable corneal data in a corneal peripheral area. It has been proved that shape of this stimulator profile depends on assumed corneal geometry. The ellipsoidal corneal shape has been assumed. The results show that the surface on which Placido rings are painted should have a cigar-like shape. Probably none of the devices used currently in clinical practice have such shape.

1. Introduction

Corneal topography is currently one of the most important examinations whose object is to define the refractive properties of the eye. It is a source of detailed knowledge of the shape of the anterior surface of the cornea, which enables an early detection and diagnosis of corneal diseases (keratoconus), a vision disorders definition and also an early contact lens fitting and assessing the after-effects of wearing contact lens.

The history of the corneal topography begins in the XVII century when Christopher Scheiner [1], a German Jesuit, compared images created by reflecting from the cornea with images created by reflecting from several balls of known curvature. The next step in the corneal examination was made by Helmholtz who invented the ophtalmometer. This instrument enabled him to observe of reflection of a pair of objects placed at the known distance from the eye. In 1889 Javal modified this device in such a way that objects, being the sources of light, could rotate around the axis. This step made it possible to do measurements of minimal and maximal radii of corneal curvature and consequently to assess the corneal astigmatism [2].

In the end of the XIX century Placido, instead of a couple of small objects, used a round disc with coaxial black-white circles (so-called Placido rings or Placido discs) and a centrally situated hole through which a researcher could observe images created by reflecting from the surface of the cornea. Changes in the shape of the cornea induced certain deformations of the image of reflected rings. This innovation not only eliminated movements of the device so that a physician could measure the radii of curvature in different directions but also enabled an assessment of the corneal geometry in a larger zone than the apex of the cornea [1].

Further a photo camera was used and later an very modern CCD elements which permitted to register Placido rings images. In the course of the last few decades man has engaged in the examination a computer, which can perform huge amounts of calculations in a very short time. In this way a videokeratometer was created – computer assisted video keratometer (CAVK). This kind of keratometry is currently the most exact tool providing corneal geometry. It made it possible to obtain objective and very reliable results. Their analyse resulted in refractive surgery development [3].

Many of the devices currently used in clinical practise rely on Placido rings principle. Several studies [4]–[7] which used surfaces with known curvatures have shown that the accuracy of these instruments decreases in a peripheral zone and when the surface differs greatly from a sphere. This feature is directly combined with the shape of the surface on which Placido rings are painted.

2. Description of the problem

One of the most important problems which appear in contemporary videokeratometry is the profile of the surface on which Placido rings are painted. It is this surface geometry that determines how large area of the cornea is enlightened and diagnosed; moreover topography measurement precision depends on this geometry. It would be ideal if the image of each Placido ring was placed at the same distance from the corneal apex, so that the image plane would be formed. Figure 1 shows images formation by reflection of rays coming from a flat stimulator (in example disc) at a spherical (for simplicity) convex mirror.

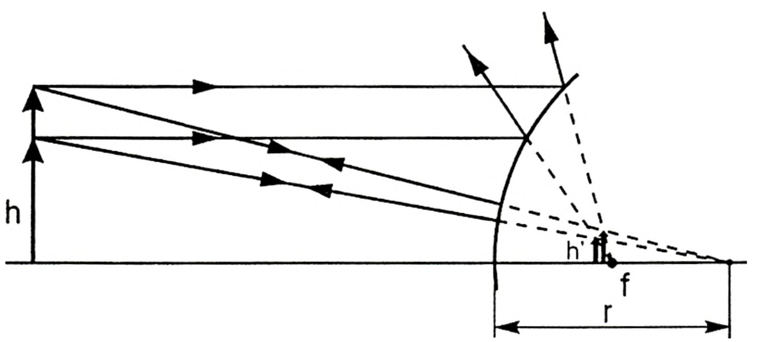

Fig. 1. Scheme of image formation of rings placed on a flat illuminator.

The up-directed arrows, placed before the mirror, represent Placido rings whose radii suit the arrows height. As you can see, the image of each ring is created at a different distance from the mirror apex, so the images form the image surface, not the plane. Objective lenses of videokeratometers, with regard to the necessity of having a large focal distance (70–100 mm), must have small depth of sharpness. Therefore, if we focus the system of such objective lens on one of the images, then information about the second image becomes blurred and no longer is reliable. In order to obtain an image less "smashed". Dekking introduced a cone-shaped illuminator which found a number of industrial applications [3]. However, as it will be shown later, in order to have a flat image of all rings, the stimulator should have the shape of a cigar. JONGSMA [3] mentions such illuminator, but it was hard to find materials describing this stimulator more precisely.

Illuminators of contemporary commercial videokeratometers are cone-shaped (Tomey TMS-1 [8], Eyesys 2000 Corneal Analysis System [9]), cylinder-shaped (Tomey TMS-3 [10]) or ellipsoid-shaped. But they do not guarantee the optimal image reconstruction. As it will be shown, the shape of an illuminator profile depends on the topography of corneal surface. Therefore we have a paradox – in order to assess the shape of the anterior corneal surface we have to use an illuminator of such geometry which guarantees formation of all Placido rings at one plane. On the other hand, in order to predict this shape of the illuminator, we have to know the corneal topography first. The purpose of this study is to describe such a keratometrical stimulator, which could give the image as flat as possible, which could perform the optimal image reconstruction.

3. Method

In the first place we assume that the thickness of tear film covering the cornea is constant all over the cornea, because disorders in this thickness cause difficulties in the videokeratometric image interpretation. Let us assume that light coming from the illuminator does not reflect at inner corneal layers. All of the rays reflect from the frontal part of the cornea which is covered by tear film.

The next assumption relates to the shape of the cornea, which is the basis of calculation of illuminator geometry. Various authors describe geometry of this part of the eye in very different ways. The most frequent approach are conic surfaces [11]–[13]. One of these surfaces is ellipsoid, given by function [14]

$$z(x, y) = \frac{R_0}{1-\varepsilon^2}\left(1-\sqrt{1-\frac{1-\varepsilon^2}{R_0^2}(x^2+y^2)}\right) \tag{1}$$

where R_0 is the apical radius of curvature of the anterior corneal surface, and ε is the ellipsoid eccentricity. Additionally, if we assume that the ray coming from the defined Placido ring at one meridian then it is reflected by the anterior corneal surface and goes to the same meridian of the entrance pupil of a keratometric objective. This means that, if we assume that each ring consists of points, then the images of those points will be formed just in the same plane as the points themselves. Therefore we can simplify the Eq. (1) to the two-dimensional problem

$$z(x) = \frac{R_0}{1-\varepsilon^2}\left(1-\sqrt{1-\frac{1-\varepsilon^2}{R_0^2}x^2}\right) \tag{2}$$

The centre of the co-ordinates system is placed at the apex of the cornea, just like in paper [14].

The next important assumption is that the images of each point (ring) are formed in the very same plane. Let this plane be the paraxial image plane, calculated for the above parameters from the formula

$$s_p = \frac{R_0}{2 + \dfrac{R_0}{|s|}} \tag{3}$$

where s is the distance from the objective main plane of device objective-lens. Let image heights vary regularly.

Calculations rely on inverse tracing of several rays. Figure 2 shows the scheme of the rays course. The rays described by the direction cosines (l_i, m_i) come out of the objective entrance pupil, placed at the distance s from the corneal apex. They converge at the image point (x_h, z_h), where z_h is the paraxial image distance s_p. On their way they come across the cornea given by the Eq. (2). The cornea, playing the role of a convex mirror, reflects these rays at the points (x_{p_i}, z_{p_i}). It is obvious that there must

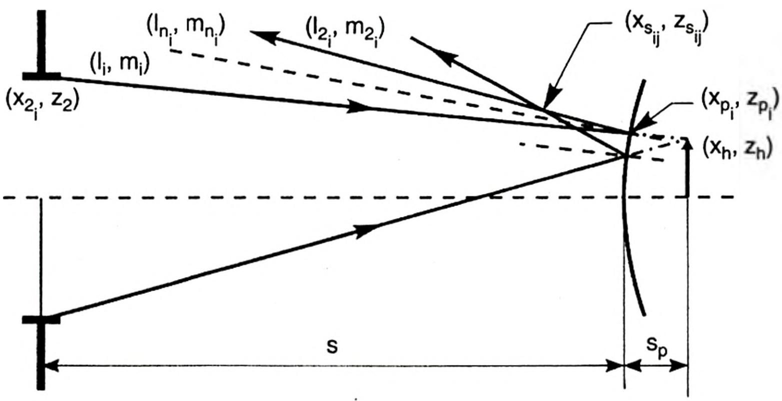

Fig. 2. Scheme of reverse ray tracing, being the basis of calculation.

be the Snell's law fulfilled. The reflected rays of direction cosines (l_{2_i}, m_{2_i}) converge and intersect at the point $(x_{s_{ij}}, z_{s_{ij}})$. This point should belong to the illuminator profile on which Placido rings are painted. This procedure should be performed for different image point height values with constant increment. It is the matter of constant increment of Placido rings images heights.

4. Results

Calculations were performed for the average normal cornea whose parameters are: $R_0 = 7.8$ mm [15], eccentricity $\varepsilon = 0.7$ [16]. Rays get into the objective lens entrance

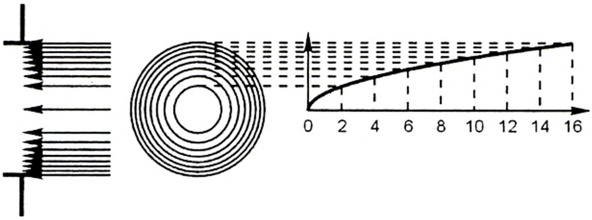

Fig. 3. Scheme explaining the configuration of getting rays into the objective entrance pupil.

pupil in the way shown in Fig. 3. This configuration suits the following situation: if one assumes that each ray brings the same quantity of energy, so the same quantity of energy falls on each unit of the entrance pupil area. That is why the rays density changes with the radius of the entrance pupil. For these values and parameters we performed calculations quoted in the previous section. Results of these calculations for each image height are two arrays describing a cloud of points $(x_{s_{ij}}, z_{s_{ij}})$ (see Fig. 4). It occurs because the cornea as an ellipsoidal mirror is not free from aberrations. In order to have one quantity describing the illuminator point, the center of cloud gravity was taken. We performed calculations for the image height from 0.2 to 5 mm, each 0.1 mm. Therefore we received 49 points composing the geometry of the illuminator. The next step was to approximate these points by the least squares method. As the result of the approximation the sixth degree polynomial $x = f(z)$ was accepted. This polynomial approximates data points very well (Fig. 4). Table 1 lists its coefficients. This polynomial describes the keratometrical stimulator geometry. If we take into consideration the axial symmetry, its shape will remind a cigar. As you can see, it is very complex geometry, more difficult to describe than conic or cylindrical surfaces used in existing clinical devices.

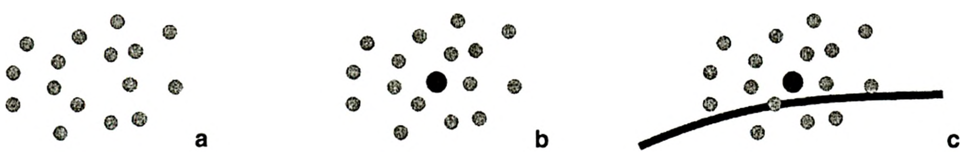

Fig. 4. Successive stages of the calculation of the illuminator surface on which Placido rings would be painted: cloud of points (**a**), getting the cloud centre of gravity (**b**), approximation by polynomial (**c**).

Table 1. Coefficients of the sixth-degree polynomial describing the surface of the illuminator.

$\times z^6$	-1.06722×10^{-9}
$\times z^5$	-2.20152×10^{-7}
$\times z^4$	-1.79544×10^{-5}
$\times z^3$	-7.37199×10^{-4}
$\times z^2$	-2.04619×10^{-2}
$\times z$	-0.52510
$\times 1$	8.86814

Table 2. Coefficients of the sixth-degree polynomials describing the illuminator geometry for different values of the ellipsoidal eccentricity.

	$\varepsilon_1 = 0.65$	$\varepsilon_2 = 0.7$	$\varepsilon_3 = 0.75$
$\times z^6$	-1.24948×10^{-9}	-1.06722×10^{-9}	-9.83035×10^{-10}
$\times z^5$	-2.62518×10^{-7}	-2.20152×10^{-7}	-2.01468×10^{-7}
$\times z^4$	-2.17431×10^{-5}	-1.79544×10^{-5}	-1.63265×10^{-5}
$\times z^3$	-8.99150×10^{-4}	-7.37199×10^{-4}	-6.66435×10^{-4}
$\times z^2$	-2.39538×10^{-2}	-2.04619×10^{-2}	-1.86938×10^{-2}
$\times z$	-0.56946	-0.52510	-0.49488
$\times 1$	8.75569	8.86814	8.54451

Next, calculations for different ellipsoid profiles were performed. Indispensable assumptions are analogous to those applied previously. We accepted several values of the eccentricity of the ellipse describing the cornea: $\varepsilon_1 = 0.65$, $\varepsilon_2 = 0.7$ $\varepsilon_3 = 0.75$. The results of calculations are the sixth-degree polynomial coefficients. They are listed in Tab. 2. Figure 5 shows plots of this polynomial.

Next, the central radius of corneal curvature was changed. Calculations were performed for several values of the radius: $R_{01} = 6.0$ mm, $R_{02} = 7.8$ mm and $R_{03} = 8.3$ mm. The results of calculations are listed in Tab. 3. Figure 6 illustrates plots of polynomials.

Table 3. Coefficients of the sixth-degree polynomials describing the illuminator geometry for different values of the apical radius of ellipsoidal cornea.

	$R_{01} = 0.65$	$R_{02} = 0.7$	$R_{03} = 0.75$
$\times z^6$	-6.29797×10^{-10}	-1.06722×10^{-9}	-1.32347×10^{-9}
$\times z^5$	-1.31194×10^{-7}	-2.20152×10^{-7}	-2.79248×10^{-7}
$\times z^4$	-1.13503×10^{-5}	-1.79544×10^{-5}	-2.31266×10^{-5}
$\times z^3$	-5.28200×10^{-4}	-7.37199×10^{-4}	-9.47618×10^{-4}
$\times z^2$	-1.79105×10^{-2}	-2.04619×10^{-2}	-2.43970×10^{-2}
$\times z$	-0.51126	-0.52510	-0.55416
$\times 1$	6.81638	8.86814	9.10313

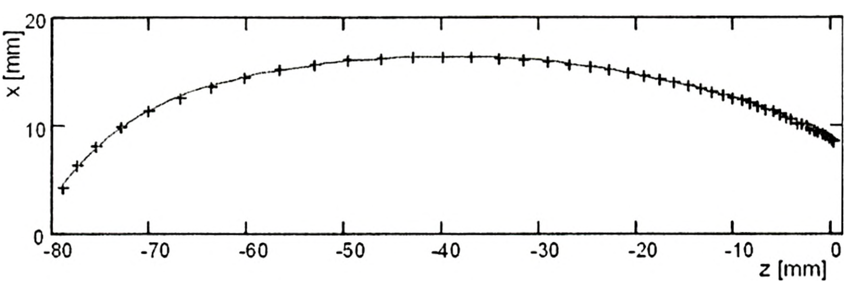

Fig. 5. Approximation of the illuminator surface by the use of the sixth degree polynomial (+++ points being the center of gravity of each "cloud", —— least-squares fitting curve).

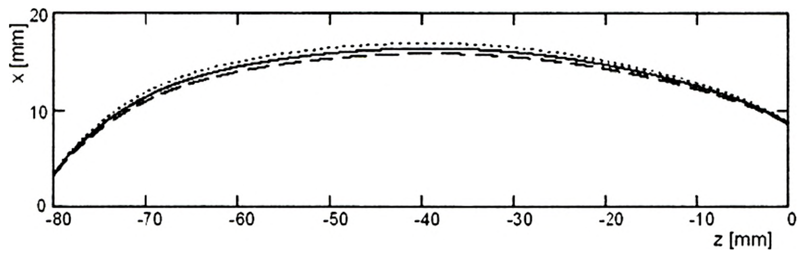

Fig. 6. Influence of the eccentricity of the ellipsoidal cornea on an illuminator shape (---- for $\varepsilon_1 = 0.65$, —— for $\varepsilon_2 = 0.7$, – – – for $\varepsilon_3 = 0.75$).

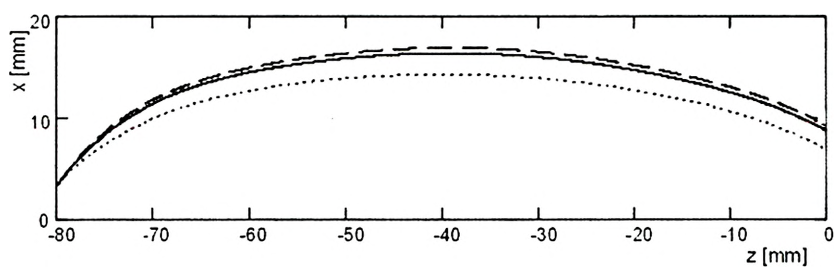

Fig. 7. Influence of the apexial curvature radius of the ellipsoidal cornea on an illuminator shape (---- for $R_{01} = 6.0$, —— for $R_{02} = 7.8$, – – – for $R_{03} = 8.3$).

As one can see, the coefficients of the polynomials describing the geometry of keratometric stimulators for various cornea profiles differ considerably. In Figure 5 one can see two nodal points to which individual plots for individual values of eccentricity converge. However, in Fig. 7 there is only one such point. It is connected with the fact that for corneas with different central radii of curvature the paraxial image appears at different distances from the apex according to Eq. (3). However one feature remains common – the cross-section of the illuminator surface on which Placido rings are painted should be cigar-shaped. Probably none of the devices used in clinical practice has a stimulator with such geometry.

6. Summary

Most of current videokeratometers based on Placido rings do not guarantee the registration of sharp images at the whole field of vision. What is more, in the case of blurred images of some Placido rings there appears an error of magnification. This error influences directly the calculated accuracy of distribution of the corneal curvature. This problem can be partially overcome by the use of imaging lens with large depth of sharpness. However, such lens does not give the correct data of magnification.

Geometry of the illuminator, which was determined in this paper, is described by the sixth-degree polynomial and is illustrated in Fig. 8. As it was proved, the shape of

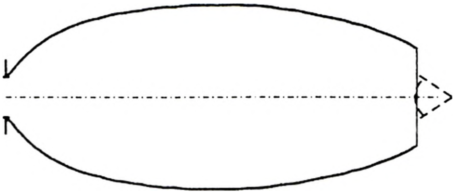

Fig. 8. Final illuminator surface shape on which Placido rings would be painted (the cross-section).

the cornea influences significantly this geometry. Then the coefficients of the polynomials vary in a remarkable range.

Each cornea is different. Keratometric examinations are performed mostly in order to assess the geometry of all types of corneas. Therefore in calculations of illuminator surface giving flat image we took into consideration "statistical" corneal geometry, according to literature, average in the population. Therefore the videokeratometer with the illuminator of the shape presented in this paper would enable registration of almost flat images in the whole field of vision of the device and therefore receiving reliable (without a magnification error) information on corneal geometry in a much larger area than by using existing topographers. This is very important, in particular, in contact lens fitting, where patients' comfort depends on good contact lens adjustment to the corneal surface.

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Received September 10, 2002
in revised form January 8, 2003