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Informational representation of fundamental phenomena in the digital electro-optical imaging devices

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This article contains basic theoretical results of mathematical interpretations of some fundamental phenomena in the modern digital electronic (electro-optical) imaging devices (systems) with pixel photodetection structure mainly from the standpoint of the mean mutual information. For that purpose, the systems in question are approximated by a suitable linear and isoplanatic signal transfer model of a 2D stationary and ergodic continuous stochastic distribution of the object scene -light intensity which is limited in size by the input field of view. The mathematical modelling of the phenomena presented is based on utilizing the continuous linear signal transfer theory and the extended modern communication and information theory for the spectral domain of spatial frequencies, in particular. The equations put forward are acceptable for an analysis and optimal synthesis of digital electronic imaging systems fitting the signal transfer model proposed. Conventionally, these systems give better information efficiency and image quality in comparison with the older analog imaging systems.

Keywords: digital imaging system, information entropy, mean mutual information.

1. Introduction

The digital (sampled) electronic (electro-optical) imaging systems (digital video -cameras) under consideration handled for example in publications [1]–[5], contain a pixel photodetection layer and produce optical imaging by means of electronic digitizing of image signals. They give conventionally a better image quality (*i.e.*, fidelity, sharpness and cleanness) and information efficiency in comparison with the classic analog imaging systems (analog video-cameras).

A continuous/discrete/continuous (c/d/c) model is the correct choice for a comprehensive analysis of modern digital electronic imaging systems. An example of such a model is introduced and described in the paper [4]. It is a linear and isoplanatic signal transfer model with a continuous input (a continuous 2D distribution of the light intensity extended from an object scene), a continuous-to-discrete conversion of the optical image signal to its sampled (discrete) electrical form, discrete quantizing and encoding (digital processing) of this form and with a discrete-to-continuous conversion (restoration) of the obtained digital electric signal into the decoded continuous and registered output optical form (registered and recovered resultant optical image) which can be observed by a human eye producing a visual signal (conscious image). The relevant fundamental phenomena corresponding to the cascade of the aforementioned successive steps are called: image gathering (sampled optical-to-electrical scanning of the object scene by pixels), image digital processing, image restoration, image display (image registration), image eye observation (image visual inspection). These phenomena for the predetermined image signal are mathematically interpreted in publications [1]–[8]. The mathematical modelling and analysis of such phenomena for stochastic (random) image signals, connected with their informational aspect, are introduced in [4], [9], [10]. A sufficiently wide and complete interpretation of these signals especially with respect to their probability, self-correlation, self-covariance, information entropy and mean mutual information in the continuous spatial and spectral domain of spatial frequencies is included in the paper [11]. The main purpose of this article is to summarize, interpret and evaluate some basic spectral mathematical relations from the last publication and also to present examples of the information-theoretic assessment of the image gathering step of digital electronic imaging systems, which are represented by the reasonable signal transfer model.

2. Signal transfer model and its basic characteristic quantities

Figure 1 depicts a spectral form of the alternative of the proposed linear and isoplanatic (spatially invariant) signal transfer model of a digital electronic imaging system with a pixel photodetection structure that is analyzed in this article. The distributed, spatially stationary and ergodic stochastic input field of view is of finite area. Such a signal (input radiance field) is represented by the optical intensity of spatial fluctuations $s_0'(x_1, x_2)$ and corresponding continuous aperiodic Fourier spectrum $\tilde{s}_0'(\mu_1, \mu_2)$ which are responsible for the following statistical and informational considerations. This first step in the operation of IG denotation in Fig. 1 accounts for perturbations due to blurring, aliasing and photodetection noise of its output (acquired) image optical signal of spatial fluctuations $s_3'(x_1, x_2)$ and of their continuous periodic Fourier spectrum $\tilde{s}_3'(\mu_1, \mu_2)$. The reference orthogonal system of linear nondimensional coordinates (x_1, x_2) is fixed relative to the object scene and relates to the pixel photodetection layer of the image gathering device (acquisition subsystem) IG. Distances in this coordinate system are measured relative to the $\xi_1 \times \xi_2$ interdetector (intersample) discrete distance with the detector (pixel) centers at the grid points with the nondimensional serial discrete pixel coordinates (μ_1, μ_2) of the values within the field of view considered, as indicated by the dots [5], [6]. Consistent with this convention, the corresponding nondimensional spatial frequencies (μ_1, μ_2) are

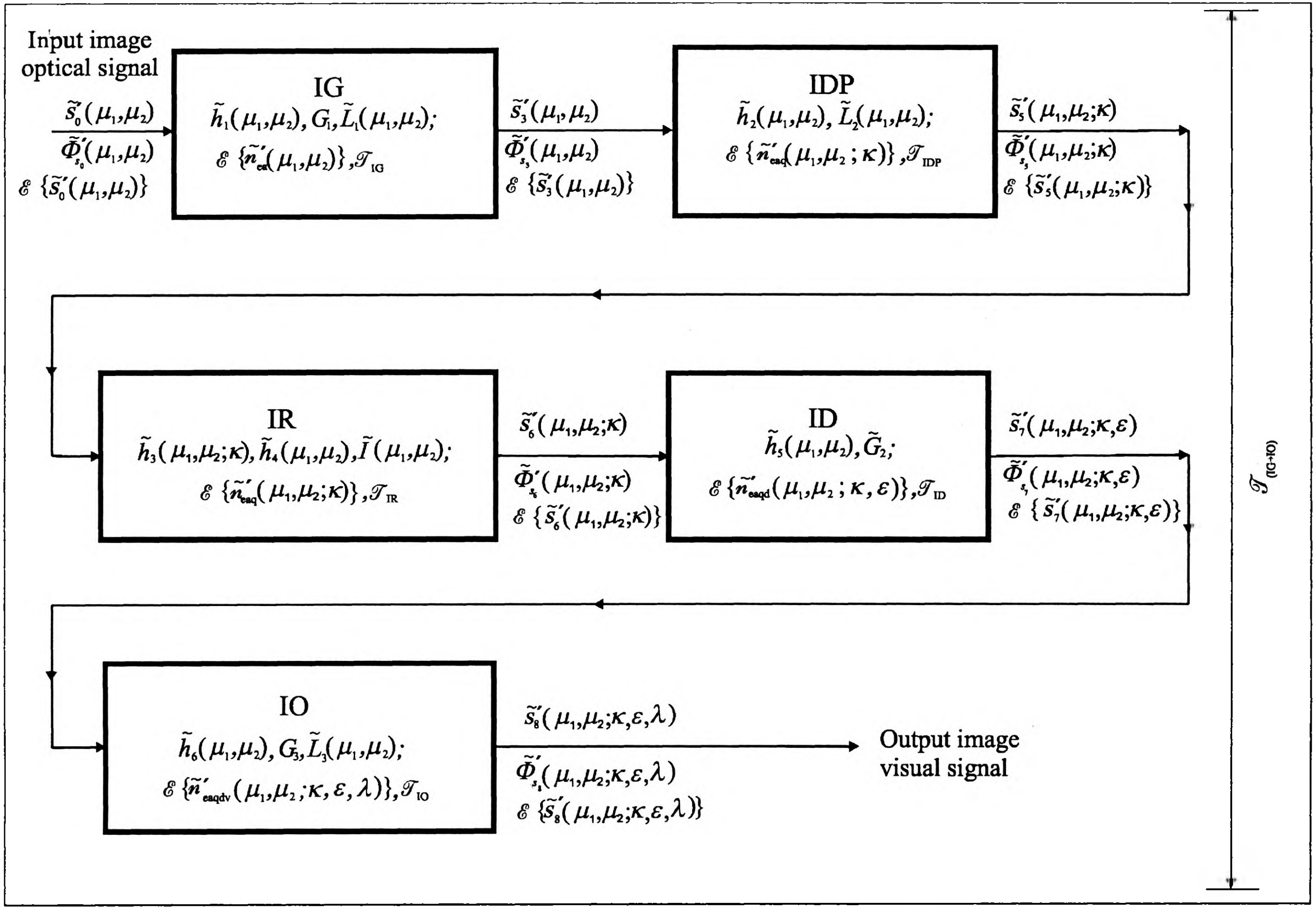

Fig. 1. Proposed signal transfer model of a digital electronic imaging system.

measured in harmonic cycles per units of the pixel sampling period (interval) ξ_1 and ξ_2 (per sample). So that the sampling frequencies are $1/\xi_1$ and $1/\xi_2$ in the vertical and horizontal directions, respectively, and the corresponding Nyquist spatial frequencies are $1/2\xi_1$ and $1/2\xi_2$. These cut-off frequencies define the so-called Nyquist sampling frequency passband or central sampling frequency passband

$$B_s = B_s\left[|\mu_1| \leq \frac{1}{2}, |\mu_2| \leq \frac{1}{2}\right] \tag{1}$$

of the aperiodic spectrum $\tilde{s}_0'(\mu_1, \mu_2)$ or of the extensive periodic spectrum $\tilde{s}_3'(\mu_1, \mu_2)$ of the sampled image optical signal $s_3'(x_1, x_2)$, respectively. Simultaneously, the insignificant values of $\tilde{s}_0'(\mu_1, \mu_2)$ can be neglected. Thus only the representation frequency passband

$$B_r = B_r[|\mu_1| \leq \eta_1, |\mu_2| \leq \eta_2] \tag{2}$$

of the spectrum $\tilde{s}_0'(\mu_1, \mu_2)$ with suitable cut-off representation frequencies (η_1, η_2) is under consideration in such a case [2]–[7]. The sampling frequency passband has a unit area, *i.e.*, $|B_s| = 1$, whereas the area of the representation frequency passband is $|B_r| = 4\eta_1\eta_2$.

In the only spectral domain (μ_1, μ_2) considered, the subsystem IG can be characterized by the deterministic spectral optical transfer function $\tilde{h}_1(\mu_1, \mu_2)$ of its input (scanning) objective (lens), by the ideal deterministic periodic spectral form

$$\begin{aligned} \tilde{L}_1(\mu_1, \mu_2) &= \sum_{(n_1, n_2)}\sum \delta(\mu_1 - n_1, \mu_2 - n_2) \\ &= \delta(\mu_1, \mu_2) + \sum_{(n_1 \neq 0, n_2 \neq 0)}\sum \delta(\mu_1 - n_1, \mu_2 - n_2) \end{aligned} \tag{3}$$

of the photodetection sampling (pixel) lattice $L_1(x_1, x_2)$ and by the steady-state gain G_1 of the linear optical radiance-to-electrical signal conversion. Simultaneously, the last sum in Eq. (3) accounts for the sampling frequency sidebands $(n_1 \neq 0, n_2 \neq 0)$.

The continuous radiance field Fourier transform $\tilde{s}_0'(\mu_1, \mu_2)$ of the continuous input (object) stochastic signal $s_0'(x_1, x_2)$ is now understood as a realization (representative) of the corresponding stationary and ergodic continuous input stochastic process $\tilde{\mathscr{S}}'_0(\mu_1, \mu_2)$. A similar meaning holds for the continuous periodic output Fourier spectrum $\tilde{s}'_3(\mu_1, \mu_2)$. By analogy, the continuous Fourier spectrum of the operated additive and discrete photodetector electric noise fluctuations $n'_e(x_1, x_2)$, which are included in the spectrum $\tilde{s}'_3(\mu_1, \mu_2)$, is denoted by the symbol $\tilde{n}'_e(\mu_1, \mu_2)$. Similarly, the continuous Fourier spectrum of the operated aliased noise in the signal (*i.e.*, the aliased signal $n'_a(x_1, x_2)$ that an insufficient sampling folds into the sampling frequency passband (1)) holds the notation $\tilde{n}'_a(\mu_1, \mu_2)$. The sum

$$\tilde{n}'_{ea}(\mu_1, \mu_2) = \tilde{n}'_e(\mu_1, \mu_2) + \tilde{n}'_a(\mu_1, \mu_2) \tag{4}$$

then represents the continuous Fourier spectrum of the image gathering total noise.

For further considerations, more acceptable representations of the spectra $\tilde{s}'_0(\mu_1, \mu_2)$, $\tilde{s}'_3(\mu_1, \mu_2)$, $\tilde{n}'_e(\mu_1, \mu_2)$, $\tilde{n}'_a(\mu_1, \mu_2)$ are the power spectral densities $\tilde{\Phi}'_{s_0}(\mu_1, \mu_2)$, $\tilde{\Phi}'_{s_3}(\mu_1, \mu_2)$, $\tilde{\Phi}'_{n_e}(\mu_1, \mu_2)$, $\tilde{\Phi}'_{n_a}(\mu_1, \mu_2)$ (*i.e.*, the Fourier spectra of the corresponding self-covariance functions [11]) and also the spectral information entropies (spectral mean own information) $\mathscr{E}\{\tilde{s}'_0(\mu_1, \mu_2)\}$, $\mathscr{E}\{\tilde{s}'_3(\mu_1, \mu_2)\}$, $\mathscr{E}\{\tilde{n}'_e(\mu_1, \mu_2)\}$, $\mathscr{E}\{\tilde{n}'_a(\mu_1, \mu_2)\}$. By means of these spectral quantities and the probability densities $p[\tilde{s}'_0(\mu_1, \mu_2)]$, $p[\tilde{s}'_3(\mu_1, \mu_2)]$, $p[\tilde{n}'_e(\mu_1, \mu_2)]$, $p[\tilde{n}'_a(\mu_1, \mu_2)]$ of the spectra $\tilde{s}'_0(\mu_1, \mu_2)$, $\tilde{s}'_3(\mu_1, \mu_2)$, $\tilde{n}'_e(\mu_1, \mu_2)$, $\tilde{n}'_a(\mu_1, \mu_2)$ it is possible to define the corresponding spectral mean mutual information by the relation [4]

$$\mathscr{T}_{IG}\{\tilde{s}'_0(\mu_1, \mu_2), \tilde{s}'_3(\mu_1, \mu_2)\} = \mathscr{E}\{\tilde{s}'_3(\mu_1, \mu_2)\} - \mathscr{E}\{\tilde{n}'_{ea}(\mu_1, \mu_2)\} \tag{5}$$

where

$$\mathscr{E}\{\tilde{s}'_3(\mu_1, \mu_2)\} = -\int_{-\infty}^{\infty} p[\tilde{s}'_3(\mu_1, \mu_2)] \log_2 p[\tilde{s}'_3(\mu_1, \mu_2)] \, \mathrm{d}[\tilde{s}'_3(\mu_1, \mu_2)], \tag{6}$$

$$\mathscr{E}\{\tilde{n}'_{ea}(\mu_1, \mu_2)\} = -\int_{-\infty}^{\infty} p[\tilde{n}'_{ea}(\mu_1, \mu_2)] \log_2 p[\tilde{n}'_{ea}(\mu_1, \mu_2)] \, \mathrm{d}[\tilde{n}'_{ea}(\mu_1, \mu_2)]. \tag{7}$$

The integrated form (information rate) of Eq. (5) over the sampling frequency passband (1), which is mainly considered in this article, can be expressed by the relations [4], [9]–[11]

$$\mathscr{T}_{\mathrm{IG}} \triangleq \mathscr{T}_{\mathrm{IG}}(B_s) = \frac{1}{2} \iint_{(B_s)} \mathscr{T}_{\mathrm{IG}}\{\tilde{s}_0'(\mu_1, \mu_2), \tilde{s}_3'(\mu_1, \mu_2)\}\, \mathrm{d}\mu_1\, \mathrm{d}\mu_2$$

$$= -\frac{1}{2} \iint_{(B_s)} \left\{ \int_{-\infty}^{\infty} p[\tilde{s}_3'(\mu_1, \mu_2)] \log_2 p[\tilde{s}_3'(\mu_1, \mu_2)]\, \mathrm{d}[\tilde{s}_3'(\mu_1, \mu_2)] \right.$$

$$\left. - \int_{-\infty}^{\infty} p[\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)] \log_2 p[\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)]\, \mathrm{d}[\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)] \right\} \mathrm{d}\mu_1\, \mathrm{d}\mu_2. \tag{8}$$

The usefulness and importance of the established quantity (5) or (8) consists in the fact that it is a suitable quantitative measure of the information about the quantity $\tilde{s}_0'(\mu_1, \mu_2)$ which is transferred by the quantity $\tilde{s}_3'(\mu_1, \mu_2)$. The signal channel of a better transmission performance yields a greater quantity (8).

The second operation of IDP denotation in the signal transfer model according to Fig. 1 represents the image digital processing which contains the filtered and electrically repeated sampling (presampling), quantizing and encoding of the image electric signal $s_3'(x_1, x_2)$, whose periodically extended continuous Fourier spectrum is $\tilde{s}_3'(\mu_1, \mu_2)$. The resultant electric digital signal $s_5'(x_1, x_2; \kappa)$ of number κ of quantizing levels and of periodic continuous Fourier spectrum $\tilde{s}_5'(\mu_1, \mu_2; \kappa)$ is influenced by the quantizing noise $n_{\mathrm{q}}'(x_1, x_2; \kappa)$ of spectral form $\tilde{n}_{\mathrm{q}}'(\mu_1, \mu_2; \kappa)$. The characteristic spectral quantities of the subsystem (image digital processing device) IDP are the deterministic optional spectral transfer function $\tilde{h}_2(\mu_1, \mu_2)$ and the deterministic presampling spectrum $\tilde{L}_2(\mu_1, \mu_2)$, which is identical with $\tilde{L}_1(\mu_1, \mu_2)$. The energetic representations of spectra $\tilde{s}_3'(\mu_1, \mu_2)$, $\tilde{s}_5'(\mu_1, \mu_2; \kappa)$, $\tilde{n}_{\mathrm{q}}'(\mu_1, \mu_2; \kappa)$ are the power spectral densities $\tilde{\Phi}_{s_3}'(\mu_1, \mu_2)$, $\tilde{\Phi}_{s_5}'(\mu_1, \mu_2; \kappa)$, $\tilde{\Phi}_{n_{\mathrm{q}}}'(\mu_1, \mu_2; \kappa)$ and the useful spectral information entropies are denoted by $\mathscr{E}\{\tilde{s}_3'(\mu_1, \mu_2)\}$ and $\mathscr{E}\{\tilde{s}_5'(\mu_1, \mu_2; \kappa)\}$. Simultaneously, the third operation (*i.e.*, the image restoration IR), yielding the aperiodic spectrum $\tilde{s}_6'(\mu_1, \mu_2; \kappa)$, can be represented by the deterministic transfer function $\tilde{h}_3(\mu_1, \mu_2; \kappa)$ of the included spatially invariant improvement Wiener filter that minimizes the root mean square of the restoring operation IR and reduces the influence of the quantizing noise $n_{\mathrm{q}}'(x_1, x_2; \kappa)$ and by the deterministic spectral transfer function $\tilde{h}_4(\mu_1, \mu_2)$ of an auxiliary adaptation) filter. Both filters act in cooperation with a suitable interpolation lattice of spectrum $\tilde{I}(\mu_1, \mu_2)$ for an effective reduction of the frequency passband and then also for interactively enhancing the resultant visual quality of the image displayed. The assumed spectrum $\tilde{I}(\mu_1, \mu_2)$ has an ideal form (3), but requiring denser sampling points ($n_1' \gg n_1$, $n_2' \gg n_2$). Such interpolation suppresses the blurring and raster effects of the IR phenomenon and of the successive image display ID. However, it does not affect the blurring inherent in the image display medium whose deterministic smoothing spectral transfer function is $\tilde{h}_5(\mu_1, \mu_2)$ and whose steady-state gain of the linear electric signal-to-optical image transformation is G_2 (conventionally inverse of G_1). The perturbation due to the display medium noise of spectrum $\tilde{n}_d'(\mu_1, \mu_2; \varepsilon)$ can be characterized in terms of the medium quantizing number ε of distinguishable gray levels.

By means of the image eye observation IO depicted in Fig. 1, the displayed image of continuous aperiodic spectrum $\tilde{s}_7'(\mu_1, \mu_2; \kappa, \varepsilon)$ is transformed into the visual signal of spectrum $\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)$. This operation is characterized by the deterministic visual (observer's eye) spectral transfer function $\tilde{h}_6(\mu_1, \mu_2)$ and by the steady-state gain G_3 of the assumed linear optical radiance-to-visual signal conversion. Simultaneously, the eye observation is also influenced by the photoreceptor sampling lattice of the Fourier spectrum $\tilde{L}_3(\mu_1, \mu_2)$, which can be assumed to be analogous to the spectrum (3) for $n_1'' \gg n_1$, $n_2'' \gg n_2$, and by the quantizing number λ of discrete levels that each nerve fiber can transmit from retina to the visual cartex. If the observer is free to view this image closely, then the visual sampling frequency passband B_v of the observer's eye readily encompasses the passband (1) of the displayed image. Hence, the perturbations in viewing the image are limited only to blurring and to visual (observation, eye) noise of the spectrum $\tilde{n}_v'(\mu_1, \mu_2; \lambda)$. Similarly, with the case of $\tilde{I}(\mu_1, \mu_2)$, the eye sampling influence of $\tilde{L}_3(\mu_1, \mu_2)$ can be neglected and then the continuous spectrum $\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)$ remains aperiodic.

3. Information-mathematical modelling

From the input/output (end-to-end) point of view and the aforementioned assumptions, signal transfer system, as depicted in Fig. 1, can be characterized by the resultant signal transfer spectral relations [4], [11]

$$\begin{aligned}
&\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda) \\
&= G_3[\tilde{s}_7'(\mu_1, \mu_2; \kappa, \varepsilon)\tilde{h}_6(\mu_1, \mu_2)] \otimes \tilde{L}_3(\mu_1, \mu_2) + \tilde{n}_v'(\mu_1, \mu_2; \lambda) \\
&= G_1 G_2 G_3 \tilde{s}_0'(\mu_1, \mu_2)\tilde{H}_{(1 \to 6)}(\mu_1, \mu_2; \kappa) + \tilde{n}_{\mathrm{eaqdv}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda).
\end{aligned} \tag{9}$$

The symbol $\otimes$ represents the convolution, the product

$$\begin{aligned}
&\tilde{H}_{(1 \to 6)}(\mu_1, \mu_2; \kappa) \\
&= \tilde{h}_1(\mu_1, \mu_2)\tilde{h}_2(\mu_1, \mu_2)\tilde{h}_3(\mu_1, \mu_2; \kappa)\tilde{h}_4(\mu_1, \mu_2)\tilde{h}_5(\mu_1, \mu_2)\tilde{h}_6(\mu_1, \mu_2)
\end{aligned} \tag{10}$$

is the resultant spectral transfer function of the complete transfer system of Fig. 1, and the quantity

$$\tilde{n}'_{\mathrm{eaqdv}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda) = G_3 \tilde{n}'_{\mathrm{eaqd}}(\mu_1, \mu_2; \kappa, \varepsilon)\tilde{h}_6(\mu_1, \mu_2) + \tilde{n}'_{\mathrm{v}}(\mu_1, \mu_2; \lambda)$$

$$= G_3 \tilde{n}'_{\mathrm{eaqd}}(\mu_1, \mu_2; \kappa, \varepsilon)\tilde{h}_6(\mu_1, \mu_2) + \tilde{n}'_{\mathrm{v}}(\mu_1, \mu_2; \lambda)$$

$$= G_3\left\{\left\{G_2\left\{[\tilde{n}'_{\mathrm{e}}(\mu_1, \mu_2) + \tilde{n}'_{\mathrm{a}}(\mu_1, \mu_2)] \sum\sum_{(n_1 \neq 0, n_2 \neq 0)} \tilde{h}_2(\mu_1 - n_1, \mu_2 - n_2) + \tilde{n}'_{\mathrm{q}}(\mu_1, \mu_2; \kappa)\right\}\right.\right.$$

$$\left.\left. \times \tilde{H}_{(3\to 4)}(\mu_1, \mu_2; \kappa)\tilde{h}_5(\mu_1, \mu_2) + \tilde{n}'_{\mathrm{d}}(\mu_1, \mu_2; \varepsilon)\right\}\right\} \tilde{h}_6(\mu_1, \mu_2) + \tilde{n}'_{\mathrm{v}}(\mu_1, \mu_2; \lambda) \tag{11}$$

represents the total noise in the signal (9). Simultaneously, the Eq. (4) is valid, where

$$\tilde{n}'_{\mathrm{a}}(\mu_1, \mu_2) = G_1[\tilde{s}'_0(\mu_1, \mu_2)\tilde{h}_1(\mu_1, \mu_2)] \otimes \sum_{(n_1 \neq 0, n_2 \neq 0)}\sum \delta(\mu_1 - n_1, \mu_2 - n_2)$$

$$= G_1 \sum_{(n_1 \neq 0, n_2 \neq 0)}\sum \tilde{s}'_0(\mu_1 - n_1, \mu_2 - n_2)\, \tilde{h}_1(\mu_1 - n_1, \mu_2 - n_2). \tag{12}$$

Also the relations:

$$\sum_{(n_1 \neq 0, n_2 \neq 0)}\sum \tilde{h}_1(\mu_1 - n_1, \mu_2 - n_2) = \tilde{h}_1(\mu_1, \mu_2) \otimes \sum_{(n_1 \neq 0, n_2 \neq 0)}\sum \delta(\mu_1 - n_1, \mu_2 - n_2), \tag{13}$$

$$\tilde{H}_{(3\to 4)}(\mu_1, \mu_2; \kappa) = \tilde{h}_3(\mu_1, \mu_2; \kappa)\, \tilde{h}_4(\mu_1, \mu_2) \tag{14}$$

were utilized. The energetic (power spectral density) analogues of the relations (9) and (11) are:

$$\tilde{\Phi}'_{s_8}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)$$

$$= G_3^2 \tilde{\Phi}'_{s_7}(\mu_1, \mu_2; \kappa, \varepsilon)\left|\tilde{h}_6(\mu_1, \mu_2)\right|^2 \otimes \tilde{L}_3(\mu_1, \mu_2) + \tilde{\Phi}'_{n_{\mathrm{v}}}(\mu_1, \mu_2; \lambda)$$

$$= (G_1 G_2 G_3)^2 \tilde{\Phi}'_{s_0}(\mu_1, \mu_2)\left|\tilde{H}_{(1\to 6)}(\mu_1, \mu_2; \kappa)\right|^2 + \tilde{\Phi}'_{n_{\mathrm{eaqdv}}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda), \tag{15}$$

$$
\begin{aligned}
&\tilde{\Phi}'_{n_{\mathrm{eaqdv}}}(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda) \\
&= G_3^2\, \tilde{\Phi}'_{n_{\mathrm{eaqd}}}(\mu_1, \mu_2;\ \kappa, \varepsilon) \left|\tilde{h}_6(\mu_1, \mu_2)\right|^2 + \tilde{\Phi}'_{n_{\mathrm{v}}}(\mu_1, \mu_2;\ \lambda) \\
&= G_3^2 \Bigg\{ \Bigg\{ G_2^2 \Bigg\{ \Bigg[\tilde{\Phi}'_{n_{\mathrm{e}}}(\mu_1, \mu_2) \\
&+ G_1^2 \sum_{(n_1 \neq 0,\, n_2 \neq 0)}\sum \tilde{\Phi}'_{s_0}(\mu_1 - n_1, \mu_2 - n_2) \left|\tilde{h}_1(\mu_1 - n_1, \mu_2 - n_2)\right|^2 \Bigg] \\
&\times \sum_{(n_1 \neq 0,\, n_2 \neq 0)}\sum \left|\tilde{h}_2(\mu_1 - n_1, \mu_2 - n_2)\right|^2 + \tilde{\Phi}'_{n_{\mathrm{q}}}(\mu_1, \mu_2;\ \kappa) \Bigg\} \\
&\times \left|\tilde{H}_{(3 \to 4)}(\mu_1, \mu_2;\ \kappa) \tilde{h}_5(\mu_1, \mu_2)\right|^2 + \tilde{\Phi}'_{n_{\mathrm{d}}}(\mu_1, \mu_2;\ \varepsilon) \Bigg\} \Bigg\} \\
&\times \left|\tilde{h}_6(\mu_1, \mu_2)\right|^2 + \tilde{\Phi}'_{n_{\mathrm{v}}}(\mu_1, \mu_2;\ \lambda)
\end{aligned}
\tag{16}
$$

and at last the resultant spectral and integrated mean mutual information holds the general forms:

$$
\begin{aligned}
&\mathscr{I}_{(\mathrm{IG} \to \mathrm{IO})}\{\tilde{s}'_0(\mu_1, \mu_2), \tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)\} \\
&= \mathscr{E}\{\tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)\} - \mathscr{E}\{\tilde{n}'_{\mathrm{eaqdv}}(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)\} \\
&= -\int_{-\infty}^{\infty} \not{p}[\tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)]\ \log_2 \not{p}[\tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)]\, \mathrm{d}[\tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)] \\
&+ \int_{-\infty}^{\infty} \not{p}[\tilde{n}'_{\mathrm{eaqdv}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)] \\
&\times \log_2 \not{p}[\tilde{n}'_{\mathrm{eaqdv}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)]\, \mathrm{d}[\tilde{n}'_{\mathrm{eaqdv}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)],
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\mathscr{I}_{(\mathrm{IG} \to \mathrm{IO})} &\triangleq \mathscr{I}_{(\mathrm{IG} \to \mathrm{IO})}(B_{\mathrm{s}}) \\
&= \frac{1}{2} \iint_{(B_{\mathrm{s}})} \mathscr{I}_{(\mathrm{IG} \to \mathrm{IO})}\{\tilde{s}'_0(\mu_1, \mu_2), \tilde{s}'_8(\mu_1, \mu_2;\ \kappa, \varepsilon, \lambda)\}\, \mathrm{d}\mu_1\, \mathrm{d}\mu_2.
\end{aligned}
\tag{18}
$$

4. Examples of information-theoretical assessment

Let us consider only the IG step and also the whole input/output transfer system in Fig. 1 under a Gaussian approximation of the relevant probability densities $\not{p}[\tilde{s}_3'(\mu_1, \mu_2)]$, $\not{p}[\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)]$, $\not{p}[\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)]$ and $\not{p}[\tilde{n}_{\mathrm{eaqdv}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)]$ of similar expressions [9]:

$$\not{p}[\tilde{s}_3'(\mu_1, \mu_2)] = \frac{1}{\pi \tilde{\Phi}_{s_3}'(\mu_1, \mu_2)} \exp\left\{ - \frac{|\tilde{s}_3'(\mu_1, \mu_2)|^2}{\tilde{\Phi}_{s_3}'(\mu_1, \mu_2)} \right\}, \tag{19}$$

$$\not{p}[\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)] = \frac{1}{\pi \tilde{\Phi}_{n_{\mathrm{ea}}}'(\mu_1, \mu_2)} \exp\left\{ - \frac{|\tilde{n}_{\mathrm{ea}}'(\mu_1, \mu_2)|^2}{\tilde{\Phi}_{n_{\mathrm{ea}}}'(\mu_1, \mu_2)} \right\}, \tag{20}$$

$$\not{p}[\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)] = \frac{1}{\pi \tilde{\Phi}_{s_8}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)} \exp\left\{ - \frac{|\tilde{s}_8'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)|^2}{\tilde{\Phi}_{s_8}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)} \right\}, \tag{21}$$

$$\not{p}[\tilde{n}_{\mathrm{eaqdv}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)] = \frac{1}{\pi \tilde{\Phi}_{n_{\mathrm{eaqdv}}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)} \exp\left\{ - \frac{|\tilde{n}_{\mathrm{eaqdv}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)|^2}{\tilde{\Phi}_{n_{\mathrm{eaqdv}}}'(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)} \right\}. \tag{22}$$

In this case, the general relations (8) and (18) become [4]

$$\mathscr{T}_{\mathrm{IG}} = \frac{1}{2} \iint\limits_{(B_s)} \log_2\left[1 + G_1^2 \frac{\tilde{\Phi}_{s_3}'(\mu_1, \mu_2) |\tilde{h}_1(\mu_1, \mu_2)|^2}{\tilde{\Phi}_{n_{\mathrm{ea}}}'(\mu_1, \mu_2)} \right] \mathrm{d}\mu_1 \, \mathrm{d}\mu_2 \tag{23}$$

where

$$\tilde{\Phi}_{n_{\mathrm{ea}}}'(\mu_1, \mu_2) = G_1^2 \sum_{(n_1 \neq 0,\, n_2 \neq 0)} \sum \tilde{\Phi}_{s_0}'(\mu_1 - n_1, \mu_2 - n_2) |\tilde{h}_1(\mu_1 - n_1, \mu_2 - n_2)|^2 + \tilde{\Phi}_{n_e}'(\mu_1, \mu_2) \tag{24}$$

and

$$\mathscr{T}_{(\mathrm{IG} \to \mathrm{IO})}$$

$$= \frac{1}{2} \iint_{(B_s)} \log_2 \left[1 + (G_1 G_2 G_3)^2 \frac{\tilde{\Phi}'_{s_0}(\mu_1, \mu_2) \left| \tilde{H}_{(1 \to 6)}(\mu_1, \mu_2; \kappa) \right|^2}{\tilde{\Phi}'_{\mathscr{N}_{\mathrm{caqdv}}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)} \right] \mathrm{d}\mu_1 \, \mathrm{d}\mu_2 \tag{25}$$

where $\tilde{\Phi}'_{\mathscr{N}_{\mathrm{caqdv}}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda)$ is given by Eq. (16).

For simpler expressions of relations (23) and (25), let us also assume the white forms of the noises considered. In such a case, their variances fulfil the equalities [4], [9]–[11]:

$$\sigma^2_{\mathscr{N}_{\mathrm{ca}}} = \iint_{(B_s = 1)} \tilde{\Phi}'_{\mathscr{N}_{\mathrm{ca}}}(\mu_1, \mu_2) \, \mathrm{d}\mu_1 \, \mathrm{d}\mu_2 = \tilde{\Phi}'_{\mathscr{N}_{\mathrm{ca}}}(\mu_1, \mu_2) = \mathrm{const}, \tag{26}$$

$$\begin{aligned} \sigma^2_{\mathscr{N}_{\mathrm{caqdv}}} &= \iint_{(B_s = 1)} \tilde{\Phi}'_{\mathscr{N}_{\mathrm{caqdv}}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda) \, \mathrm{d}\mu_1 \, \mathrm{d}\mu_2 \\ &= \tilde{\Phi}'_{\mathscr{N}_{\mathrm{caqdv}}}(\mu_1, \mu_2; \kappa, \varepsilon, \lambda) = \mathrm{const} \end{aligned} \tag{27}$$

whereas the variance for the signal spectrum $\tilde{\Phi}'_{s_0}(\mu_1, \mu_2)$ can be obtained by means of the integration

$$\sigma^2_{s_0} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\Phi}'_{s_0}(\mu_1, \mu_2) \, \mathrm{d}\mu_1 \, \mathrm{d}\mu_2 \triangleq \iint_{(B_r)} \tilde{\Phi}'_{s_0}(\mu_1, \mu_2) \, \mathrm{d}\mu_1 \, \mathrm{d}\mu_2. \tag{28}$$

Simultaneously, the rates

$$(\mathrm{SNR})_{\mathscr{N}_{\mathrm{ca}}} = \left(\frac{G_1 \sigma_{s_0}}{\sigma_{\mathscr{N}_{\mathrm{ca}}}} \right)^2, \quad (\mathrm{SNR})_{\mathscr{N}_{\mathrm{caqdv}}} = \left(\frac{G_1 G_2 G_3 \sigma_{s_0}}{\sigma_{\mathscr{N}_{\mathrm{caqdv}}}} \right)^2 \tag{29}$$

can be advantageously treated as the parametric rms-signal-to-rms-noise ratios for calculations of Eqs. (23) and (25).

Typical results obtained from the calculation of theoretical expressions of the integrated mean mutual information for separate steps and also for the complete input/output cases of digital electronic imaging systems according to the signal transfer model of Fig. 1 have already been published [4], [9]–[11] with respect to the chosen various values of the corresponding parameters. In Fig. 2, only some comparative

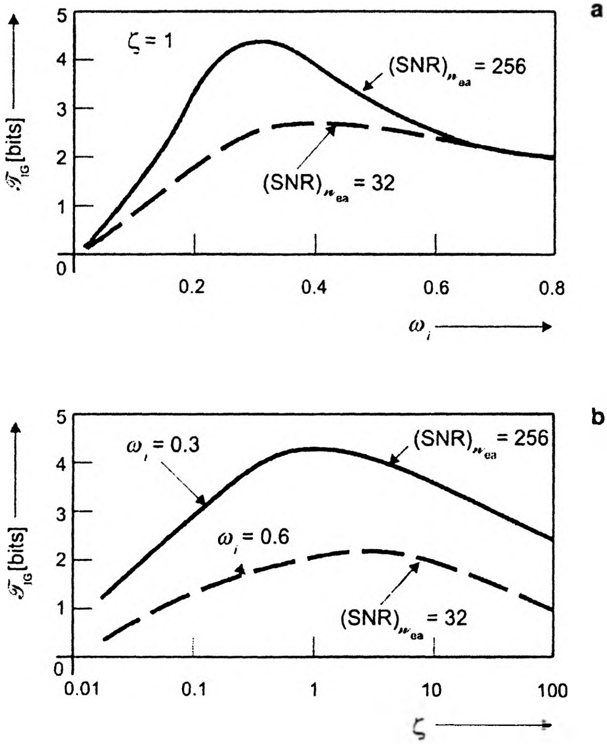

Fig. 2. Graphs of the integrated mean mutual information $\mathscr{I}_{IG}$ as a function of the variable quantity ω_i (**a**) or ζ (**b**), for the marked values of the parameter $(SNR)_{''ea}$.

graphs are shown, which relate to the simple expression (23). It is considered to be a natural stochastic object scene of a typical power spectral density [4], [12]–[15]

$$\tilde{\Phi}'_{s_0}(\mu_1, \mu_2) = \frac{2\pi\sigma_{s_0}\zeta^2}{[1 + (2\pi\zeta\omega)^2]^{3/2}} \tag{30}$$

where $\omega^2 = \mu_1^2 + \mu_2^2$. The distance parameter ζ expresses a mean spatial, detail of the stochastic object scene $s_0'(x_1, x_2)$ relative to the pixel sampling intervals (ξ_1, ξ_2) of the image gathering operation IG, which is represented by the assumed spectral transfer function of a Gaussian shape [6], [10]

$$\tilde{h}_I(\mu_1, \mu_2) = \exp\left[-\left(\frac{\omega}{\omega_i}\right)^2\right]. \tag{31}$$

The quantity ω_i is the so-called response index for which $\tilde{h}_I(\mu_1, \mu_2) = \tilde{h}_I(0, 0)/e$. The graphs in Fig. 2a refer to the quantity $\mathscr{I}_{IG}$ *vs.* ω_i for the chosen constant parameter values $\zeta = 1$ and $(\mathrm{SNR})_{n_{ea}} = 256$ or $\zeta = 1$ and $(\mathrm{SNR})_{n_{ea}} = 32$, respectively, whereas the graphs in Fig. **2b** present the dependence of $\mathscr{I}_{\mathrm{IG}}$ *vs.* ζ for $\omega_i = 0.3$ and $(\mathrm{SNR})_{n_{ea}} = 256$ or $\omega_i = 0.6$ and $(\mathrm{SNR})_{n_{ea}} = 32$, respectively [4]. These partial results testify to significant influence of the parameters considered on the quantity (23). It is also evident that the values $\zeta = 1$, $\omega_i = 0.3$ and $(\mathrm{SNR})_{n_{ea}} = 256$ (similar to human eye) give the maximum $(\mathscr{I}_{\mathrm{IG}})_{\max} = 4.4$, whereas for $\zeta = 1$, $\omega_i = 0.6$ and $(\mathrm{SNR})_{n_{ea}} = 32$ (similar to traditional TV cameras) the corresponding value of the quantity (23) is $\mathscr{I}_{\mathrm{IG}} = 2.4$.

5. Conclusions

The contribution of this communication consists in the review, interpretation and evaluation of some basic continuous spectral mathematical expressions relating to the transfer of a stationary and ergodic stochastic spatial image signal through a digital (sampled) electronic (electro-optical) imaging system. This system is represented by the proposed linear and isoplanatic signal transfer model and the considerations are mainly based on utilizing the spectral and integrated mean mutual information. The general mathematical expressions are connected with the information entropy of the signals and noises considered and with their probability densities. The spectral mean information equations introduced for the model in question are also completed by the corresponding resultant signal and power spectral density transfer relations. Besides their input/output forms, also the separate image gathering device is considered.

It is also shown that the relations presented for the integrated mean mu tual information can be modified into simpler forms by an acceptable assumption that the relevant signal and noise probability densities are Gaussian and the noises are white. The presented comparative graphs relate to this modification and separately to the image gathering step of the transfer model. Simultaneously, a theoretic natural stochastic object scenes and Gaussian shape of the gathering step spectral transfer function are assumed. The separate graphs testify to the significant dependence of the integrated mean mutual information on the chosen parameters. The more complete results for various conditions and arrangements are designed for further investigations.

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