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Numerical study of the contribution of the cornea and crystalline lens to the longitudinal chromatic aberration of the human eye

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The contribution of the cornea and the crystalline lens to the longitudinal chromatic aberration (LCA) of the human eye at different pupil diameters was investigated numerically. Two different functions were used to approximate corneal shape. Crystalline lens was represented by hundreds of shells with refractive indices increasing from the cortical to the inner shells. Lenticular shells had a form of semi-ellipsoids joined smoothly at the equator. A computer-modelling program was used to simulate refraction through the systems considered. Introducing dispersion of the elements of the eye the longitudinal chromatic aberration has been calculated. It appeared that LCA of both the cornea and the crystalline lens, and consequently, of the whole eye depends on the diameter of the input beam. Calculations showed that the crystalline lens was more dispersive than cornea.

1. Introduction

Human eye is not an ideal optical system. In the eye there exist a number of different aberrations such as the spherical and chromatic aberration, astigmatism and coma. Each of these aberrations has an influence on the optical performance of the eye. However, because of the specific structure of the retina, the off-axis aberrations are of less importance than on-axis ones.

In the study, we consider the chromatic properties of the eye's elements. All transparent substances have colour dispersion. This feature arises from the dependence between the refractive index and wavelength. In typical substances refractive index decreases with the increasing wavelength – dispersion is normal.

Chromatic aberration of the eye is the problem that arises from the dispersion of the ocular media. The eye has a substantial amount of longitudinal chromatic aberration (LCA), because short wavelengths focus in front of the retina, while long wavelengths focus farther back in the eye. Some recent experiments show that chromatic aberration might be not only a problem, but also a useful factor influencing accommodation. Ivanoff [1] suggested that there was a dependence between the chromatic aberration and accommodation. Aggarwala *et al.* [2] and Kruger *et al.* [3] showed that visual system extracts information about the defocus from the colour pattern on the retina. This information is then used by the eye to run the mechanism that drives the crystalline lens in the appropriate direction.

There are rather scarce experimental data on dispersion of the ocular media. SIVAK and MANDELMAN [4] measured dispersion of the ocular media for different vertebrate species. They found that the aqueous humour, vitreous body and cornea have dispersion similar to that of water, while the lens is more dispersive than water. Some of the authors have also measured the longitudinal chromatic aberration of the human eye [5], [6]. This value is known to be about 2.5 D. However, there are little experimental and numerical results about the contribution of the cornea and lens to the chromatic aberration of the human eye. In the paper, we investigate numerically the value of LCA in the human eye given by the cornea and the lens in the unaccommodated state. We study the dependence between pupil diameters and the amount of LCA for the cornea, the crystalline lens and the whole eye. Different approximations of the corneal shape are considered. Crystalline lens is presented in the form of hundreds of ellipsoidal surfaces with refracting index increasing from the cortical to the inner shells. The results of calculations are compared with the experimental data.

2. Corneal and lenticular shape

In the paper, we consider the numerical models of the cornea and crystalline lens which are often used in the literature.

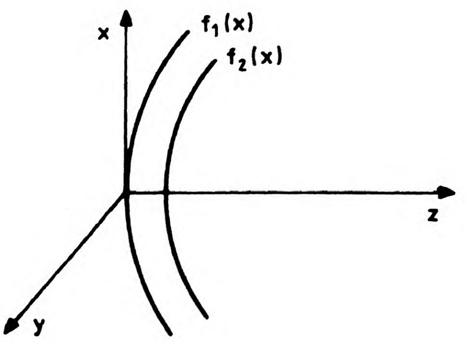

Fig. 1. Schematic presentation of the model of the cornea

Cornea is presented as a single or bilayer refracting surface (Fig. 1). We use two different functions describing corneal shape: hyperbolic cosine and parabolic. In the case of parabolic functions, these can be written as:

$$f_1(x) = \frac{x^2}{2R_{01}}, \qquad f_2(x) = \frac{x^2}{2R_{02}}, \tag{1}$$

and in the case of hyperbolic cosine functions:

$$f_1(x) = \frac{R_{01}}{3}\left[\cosh\left(\frac{\sqrt{3}}{R_{01}}x\right)-1\right], \qquad f_2(x) = \frac{R_{02}}{3}\left[\cosh\left(\frac{\sqrt{3}}{R_{02}}x\right)-1\right]+d \tag{2}$$

where R_{01} and R_{02} are the vertex radii of curvature of the first and second corneal surfaces, respectively, d is the thickness of the cornea. Both of these functions are approximations of the geometry of the cornea based, on the experimental and numerical studies of the refractive properties of the cornea and are used in the literature [7], [8].

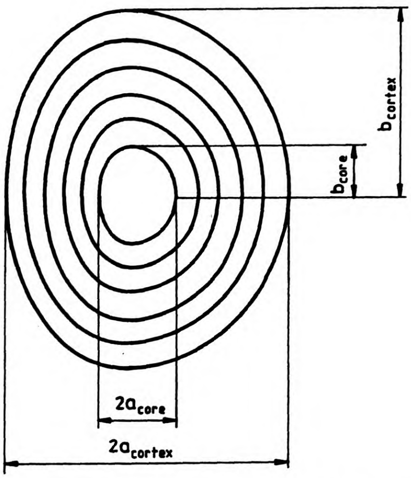

Fig. 2. Schematic presentation of the shell structure of the crystalline lens

In the sagittal section crystalline lens is described by two half-ellipses joined smoothly at the equator as presented in Fig. 2. This is in accordance with the experimental view (isoindicial curves found by JAGGER [9]) and with the modelling by other authors (SMITH *et al.* [10]). The mathematical form of the lens surfaces may be expressed as

$$2Rz = x^2+y^2+\varepsilon z^2 \tag{3}$$

where R is the vertex radii of curvature, ε – ellipticity. R and ε are related to the major and minor semi-axes by the equations:

$$R = \frac{b^2}{a}, \quad \varepsilon = \frac{b^2}{a^2}. \tag{4}$$

It is assumed in the model that lens consists of two main parts: a homogeneous core of approximately 1 mm in height (with constant refractive index) and a cortex consisting of a number of shells with the refractive index increasing from outer to inner shells. This assumption is based on the experimental observations made by POMERANTZEFF *et al.* [11]. The number of shells in the crystalline lens is assumed to be 400. This value was calculated taking into account the lens fiber thickness (6 μm) and lens geometry parameters listed in Tab. 1. The values of the parameters of the

Table 1. Geometry parameters of the cornea and crystalline lens

	Cornea Anterior	Cornea Posterior	Lens cortex Anterior	Lens core Anterior	Lens core Posterior	Lens cortex Posterior
R [mm]	7.8	6.5	10	2.5	1.5	6
ε			6.25	6.25	2.25	2.25

corneal and lenticular shape accepted in calculations are based on the Gullstrand data and are presented in Tab. 1.

At the end the optical system of the whole eye has been modelled. We have found no significant differences in chromatic properties of the cornea models considered (see the results). For this reason the cornea was approximated by two paraboloidal surfaces. The distance between the cornea and the crystalline lens was set to 3.6 mm (according to Gullstrand schematic eye).

3. Refractive indices and dispersion

We assumed that the refractive index distribution inside the crystalline lens changes according to the polynomial dependence

$$n(i) = = n_c+(n_o-n_c)\left(1-\frac{i}{N}\right)^p \tag{5}$$

where: n_o and n_c are the refractive indices of the lens cortex and core, respectively, $n(i)$ – refractive index of the i-th shell, N – number of shells, p – degree of the polynomial. The dependence between the refractive index distribution and the parameter p is shown in Fig. 3.

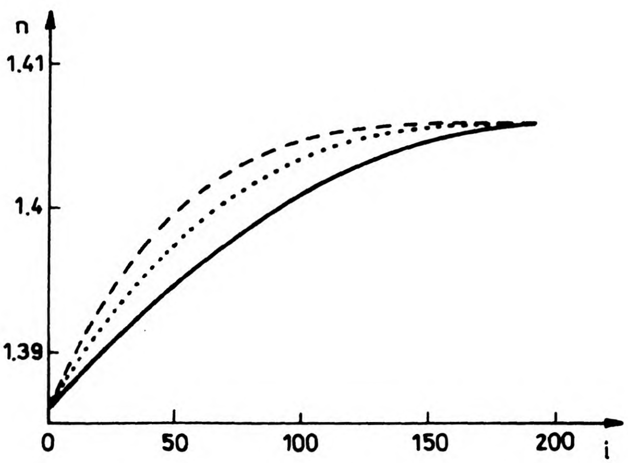

Fig. 3. Refractive index distribution within the crystalline lens according to polynomial dependence, p – polynomial degree

This equation has been applied to eye modelling by other authors [12] and is consistent with the experimental data known so far [13].

For dispersion of the cornea and the lens core and cortex we adopted Sivak and Mandelman data [4]. The data were approximated by the dispersion curve using Cornu formula

$$n(\lambda) = A + \frac{B}{C+\lambda}. \tag{6}$$

The calculated values of the parameters A, B, C for the human eye components are presented in Tab. 2.

Table 2. Values of parameters of the Cornu formula for different eye elements

	A	B	C
Cornea	1.341	0.0310	0.286
Lens cortex	1.373	0.0127	0.139
Lens core	1.337	0.0202	0.0624
Aqueous, vitreous	1.325	0.00354	0.022

4. Procedure

To simulate the ray tracing through the optical system of the eye a computer program has been written. A beam of rays parallel to the optical axis strikes the first surface of the optical system. The average distance between the rays is 0.01 mm. Each ray is traced through the optical system according to the Snell law. Rays close to the optical axis are focused in the plane F_e (Fig. 4, based on the work [14]) at a distance

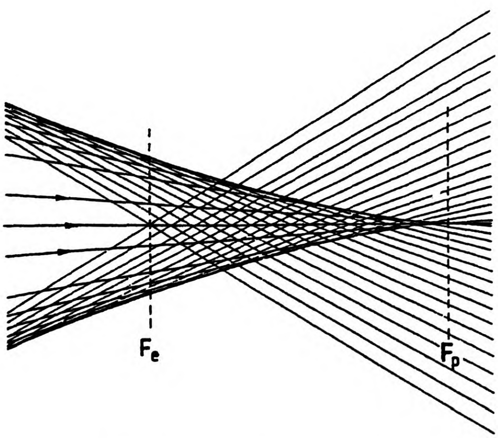

Fig. 4. Paraxial and marginal rays in the image space of the optical system

z_{max} from the first optical surface. Marginal rays, which are determined by the diameter of the beam, are focused in the plane F_p at a distance z_{min} from the first optical surface. In the program the best focal plane is found for the wavelength considered. It is assumed that this is the plane with the minimum of standard deviation taken from the spot-diagram distribution.

The calculations were made for four different wavelengths (440 nm, 486 nm, 590 nm and 650 nm), for which the appropriate refractive indices were calculated from the Cornu formula. The best focal plane location of the optical system considered changes with wavelength. The difference between the best plane locations for the red (650 nm) and blue (440 nm) wavelengths ($f_{red}-f_{blue}$) is the longitudinal chromatic aberration. To determine the relative LCA we have divided the difference $f_{red}-f_{blue}$ by the best focus for 590 nm. The given values of the chromatic aberration are expressed in percent of f_590. We have studied how the entrance beam diameter influences the LCA. We assumed that the beam diameters change from 1 mm to 3 mm (corresponding to the values of the eye's pupil diameters).

5. Results

In Table 3, the calculated values of the LCA for the cornea models considered are shown. These data are also depicted in Fig. 5. For both approximation types an increase of the beam diameter changes the amount of LCA. Chromatic aberration decreases with increasing beam diameter. A similar rate of changes is observed in all the cases. Cornea approximated by two functions gives lower LCA value than the single function approximation. The lowest value of LCA of the cornea is obtained for the approximation by two hyperbolic cosine functions and for the beam of 3 mm in diameter.

Table 3. LCA values for different cornea models expressed in percent of f_590

h [mm]	One surface cosh type	Two surfaces cosh type	One surface parabolic type	Two surfaces parabolic type
1.0	1.681	1.546	1.681	1.548
1.5	1.677	1.541	1.677	1.545
2.0	1.671	1.536	1.672	1.542
2.5	1.664	1.522	1.665	1.538
3	1.644	1.502	1.657	1.527

In the case of the crystalline lens, the first step is to find the degree of the polynomial p describing refractive index distribution for which the best focal plane position is the same as that indicated by Gulstrand data (68.5 mm calculated from the first lens surface). It is assumed that n_o and n_c are equal to 1.386 and 1.406, respectively. The best focal plane position of the lens as a function of the polynomial degree is plotted in Fig. 6. For the higher p values the best focus shifts away from the lens and the best value for the parameter p is 5. Once the degree p has been set,

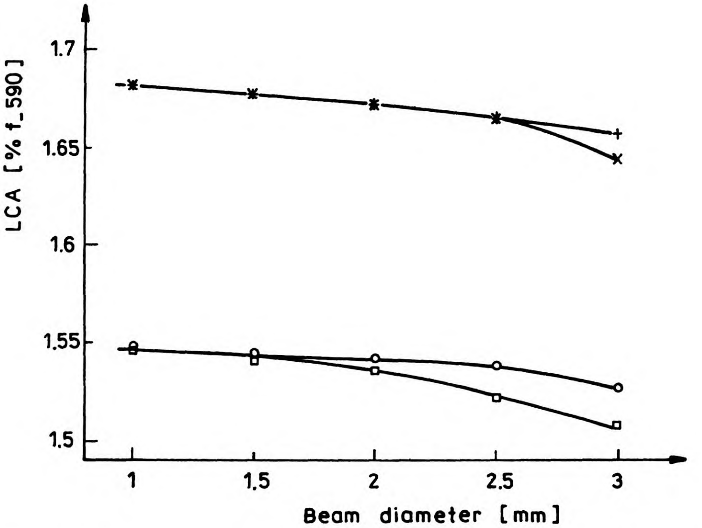

Fig. 5. LCA of the different cornea models as a function of input beam diameter

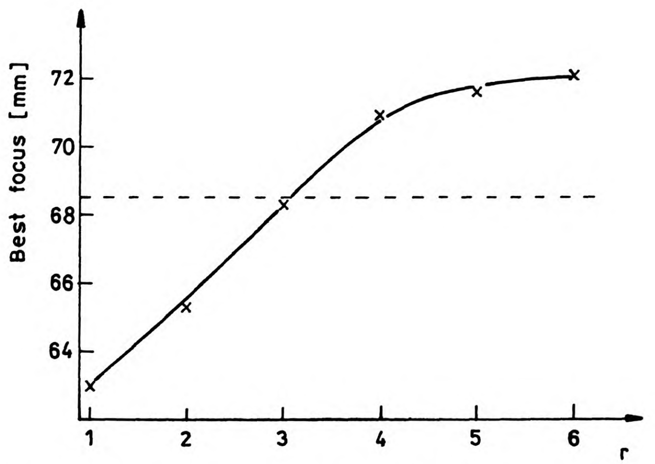

Fig. 6. Dependence between the best focal plane position of the lens and the degree of polynomial

the dispersion of the lens cortex and core is introduced. In Table 4, the values of LCA of the crystalline lens for different beam diameters are presented. These results are plotted in Fig. 7. With increasing beam diameters the value of LCA decreases. There is a small difference in LCA values for the beam diameter up to 1.5 mm. For the wider diameters (above 1.5 mm) the LCA is much lower than for the small diameters.

The next two figures present the results of calculations for the whole eye. In Figure 8, there is shown a dependence between the best focal plane position for different wavelengths as a function of beam diameter. If the beam diameter increases

Table 4. LCA values for the shell model of the crystalline lens expressed in percent of f_590

h [mm]	LCA
1.0	10.54
1.5	10.41
2.0	8.89
2.5	8.76
3	8.52

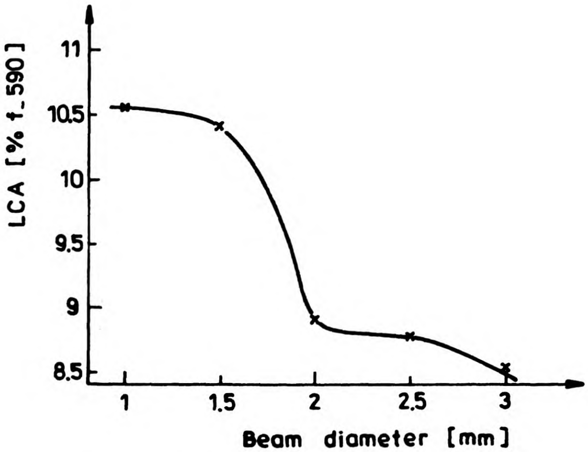

Fig. 7. LCA of the crystalline lens as a function of input beam diameter

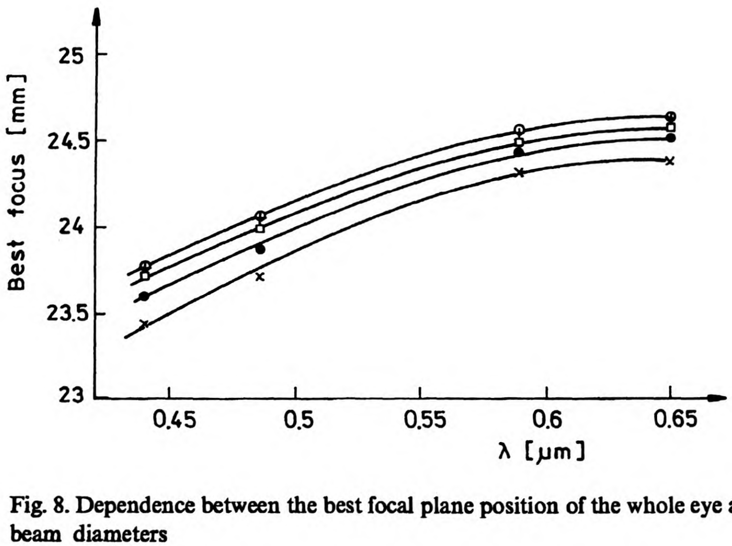

Fig. 8. Dependence between the best focal plane position of the whole eye and the wavelength for different beam diameters

Table 5. LCA values for the whole eye (cornea approximated by two paraboloidal surfaces and shell structure of the crystalline lens incorporated) expressed in percent of f_590

h [mm]	LCA
1.0	3.48
1.5	3.5
2.0	3.59
2.5	3.7
3	3.85

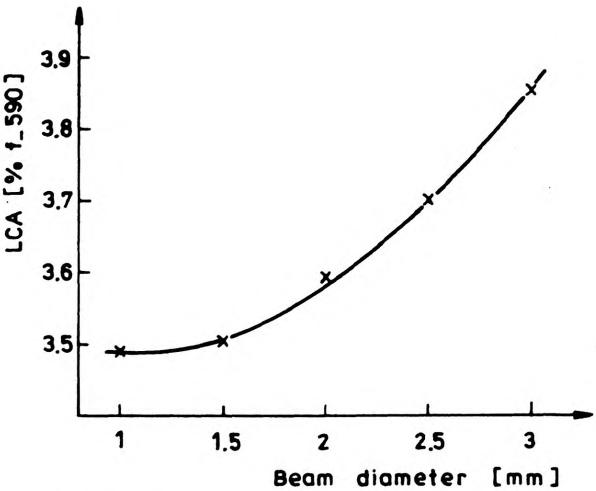

Fig. 9. LCA of the whole eye as a function of input beam diameter

the best focus shifts towards the crystalline lens for all wavelengths. In the red end of the spectrum the shift is quite small, while it becomes greater in the blue part. The changes of the LCA with increasing beam diameter are shown in Tab. 5 and in Fig. 9. The LCA of the whole eye increases with beam diameter. There is a slight difference in LCA up to the height of 1.5 mm. Above 1.5 mm one can observe greater increase of LCA than for smaller beam diameters.

6. Discussion

Taking advantage of the frequently used numerical models of the eye elements we have investigated the contribution of the cornea and crystalline lens to longitudinal chromatic aberration of human eye. The calculations showed that the LCA value of the crystalline lens is greater than that of the cornea. The LCA of the cornea differs slightly depending on whether hyperbolic cosine or parabolic approximations are performed. For both models we observe a decrease in LCA with increasing beam diameter. Cornea approximated by two functions gives lower value of LCA than approximated by a single function only. For the crystalline lens the value of LCA decreases with increasing beam diameters. This stems from the fact that lens core is more dispersive than the lens cortex. Rays which pass through the core show stronger dispersion than rays which pass only through the lens cortex. This results in greater LCA values for the smaller input beam diameters.

Two main observations can be made when modelling the whole eye. The first is that if we move outward from the optical axis, the eye becomes myopic for all wavelengths. This analytical result is consistent with the experimental observation made by WALD *et al.* [5]. The second is that up to the height of 1.5 mm from the optical axis, the LCA of the eye is almost constant compared to the wider diameters. 1.5 mm is approximate pupil diameter in daylight, and in such circumstances the colour vision is on. The wider pupil diameters appear at poor illumination, so at the conditions of scotopic vision. Then the rods are turned on, the maximum of the spectral sensitivity shifts towards the short wavelengths and we do not see colours.

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