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Hybrid aplanatic focusing lens

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In the case of simple single-element imaging system, aplanatic correction is of major importance. It can be achieved for single holographic lens, if recorded on a spherical surface. For this reason, however, high spatial frequencies have to be recorded. From technological point of view, classic combination of spherical lens with a diffraction structure of relatively low spatial frequency seems to be more advantageous. In this way, a compact optical system composed of refractive and diffractional parts is produced. In this paper, it has been shown that for convex plane hybrid lens the aplanatic correction may be achieved provided that if the object is located at infinity, the diffraction structure must be recorded on the other (plane) surface of the lens.

1. Introduction

In the contemporary instrumental optics, in addition to conventional lenses and mirrors, also the diffractive elements (diffractive lenses) produced holographically or synthetically are more and more frequently applied. Such elements were described in papers [1]–[3], for example. A natural extension of the diffractive lenses are hybrid lenses. Below, the hybrid lens is understood as a conventional glass lens with a diffraction structure deposited on one of its surfaces. Such solution, when compared to typical diffractive lenses, offers the possibility of distributing the optical power between the diffractive and refractive parts of the hybrid lens. Owing to this possibility the spatial frequencies of the diffraction structure may be relatively low which enables its performance in a typical holographic lab.

Though the hybrid lenses are recently produced in increasing quantities being used in different optical systems (*e.g.*, [4]–[6]), their imaging properties are described only partly in the literature. For example, an analysis of sphero-chromatism has been performed in [7], [8], from which it follows that due to relatively large secondary spectrum such lenses may be applied mainly on mono-chromatic light.

Aplanatic correction is important for single optical elements. An attempt to achieve such correction in the case of hybrid lens was described by the authors of the present publication in paper [9]. There they showed that for plano-convex hybrid lens with diffraction structure deposited on the first (plane) surface, the correction of coma is possible the real object is located at the distance $|z| \leqslant 2.04f'$, hence a lens focusing a parallel beam cannot be aplanatic.

The purpose of this work is to design such a lens which would enable an aplanatic correction also for the object located at infinity. Therefore, the correcting possibilities have been examined assuming that the diffraction structure is located on the second (plane) surface of the convexo-plane lens.

2. Analytical relations

In order to describe aberrations of the hybrid lenses, the most convenient method seems to be that proposed by VERBOVEN and MASAJADA [10], [11]. The notations used in the due description are explained in Fig. 1, where:

$V_{r1} = 1/r_1$ – curvature of the first refractive surface,

$V_{r2} = 1/r_2$ – curvature of the second refractive surface,

n – refractive index of the lens material,

$V_\alpha = 1/R_\alpha$ – wavefront curvature of the first wave creating the diffraction structure,

$V_\beta = 1/R_\beta$ – wavefront curvature of the second wave creating the diffraction structure,

$V = 1/z$ – reciprocity of the distance from the object point to the lens,

$V' = 1/z'$ – reciprocity of the distance from the Gaussian image to the lens.

Parameters R_α and R_β are treated as parameters describing uniquely topology of the diffraction structure.

The coefficients of third order aberrations describing the spherical aberration and coma take the form [12] (only infinitely thin lens will be considered):

$$S = -V^3+2V^2V_{r1}-VV_{r1}^2+\frac{2}{n}[V+(n-1)V_{r1}]^2(V_{r1}-V_{r2})+[V+(n-1)V_{r1}]$$
$$\times(V_{r1}^2-V_{r2}^2)+V'(V'-V_{r2})^2\mp\mu[(V_\alpha^3-V_\beta^3)-2V_{r2}(V_\alpha^2-V_\beta^2)+V_{r2}^2(V_\alpha-V_\beta)], \tag{1a}$$

$$C_y = yV\left[V^2-V'^2-VV_{r1}+\left(\frac{1}{n}V+\frac{n-1}{n}V_{r1}\right)(V_{r1}-V_{r2})+V'V_{r2}\right]. \tag{1b}$$

If the second surface is plane ($V_{r2} = 0$), these formulae are reduced to the forms:

$$S = V'^3-[V+(n-1)V_{r1}][2V+(2n-3)V_{r1}]V_{r1}-V(V-V_{r1})^2\mp(V_\alpha^3-V_\beta^3) \tag{2a}$$

and

$$C_y = yV\left[V^2-V'^2-\left(\frac{n-1}{n}\right)(V+V_{r1})V_{r1}\right]. \tag{2b}$$

When eliminating from Eq. (2b) the image position ($V' = V+\varphi$), it may be shown that the condition for coma disappearance in this case takes the form

$$\frac{1}{n(n-1)}\varphi_R^2-\frac{1}{n}V\varphi_R-\varphi(\varphi+2V) = 0 \tag{3}$$

where

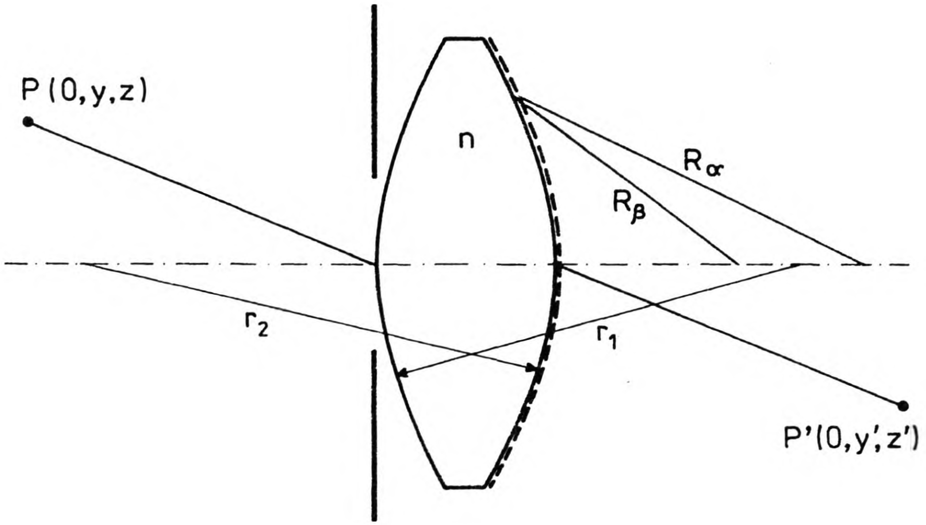

Fig. 1. Hybrid lens

$$\varphi_R = (1-n)V_{r2} \tag{4}$$

is the focusing power of the refractive part of the hybrid lens. The focusing power of the diffractive part is given by the formula

$$\varphi_D = \pm\mu(V_\alpha - V_\beta) \tag{5}$$

where $\mu = \lambda_{im}/\lambda_{rec}$ is the ratio of the wavelength of the object wave to the wavelength of the recording wave. Obviously, the total focusing power of the hybrid lens is a sum of the focusing powers of both its parts

$$\varphi = \varphi_R + \varphi_D. \tag{6}$$

In order to make Eq. (3), being quadratic with respect to φ_R, solvable the following condition must be fulfilled

$$\Delta = \frac{1}{n^2}V^2 + \frac{8\varphi}{n(n-1)}V + \frac{4\varphi^2}{n(n-1)} > 0. \tag{7}$$

Since $1/n^2 > 0$, the quadratic form (7) is positive only when:

$$V \in (-\infty, V_1] \tag{8a}$$

or

$$V \in [V_2, +\infty), \tag{8b}$$

V_1 and V_2 being solutions of Eq. (7) of the form

$$V_{1,2} = \varphi\frac{2n}{n-1}\left[-2 \pm \sqrt{\frac{3n+1}{n}}\right]. \tag{9}$$

Sign "+" corresponds to V_1 and sign "−" to V_2. If the refractive index is assumed to be $n = 1.5$, then:

$$V_1 = -23.49\varphi, \tag{10a}$$
$$V_2 = -0.52\varphi. \tag{10b}$$

This means that the correction of coma is possible, in particular, when

$$-0.52\varphi < V \leqslant 0. \tag{11}$$

In consequence, the admissible position z of the real object ranges within the interval

$$z \in [-\infty, -1.95f), \tag{12}$$

thus including the object position and infinity.

In order to find the condition for aplanatic correction, we substitute $V = 0$ to Eqs. (2a) and (2b) and equate them to zero. By eliminating V' we obtain two equations assuring aplanatism of the hybrid lens imaging the object located at infinity:

$$\pm\mu(V_\alpha^3 - V_\beta^3) = \varphi^3 \frac{(n-1)+(2-n)\sqrt{n(n-1)}}{n-1}, \tag{13}$$

$$\pm\mu(V_\alpha - V_\beta) = \varphi[1-\sqrt{n(n-1)}]. \tag{14}$$

Having solved these equations we obtain:

$$V_\alpha = \varphi\left[-\frac{B}{2} \pm \frac{1}{B}\sqrt{\frac{3}{6}(4A - B^3)}\right], \tag{15a}$$

$$V_\alpha = \varphi\left[+\frac{B}{2} \pm \frac{1}{B}\sqrt{\frac{3}{6}(4A - B^3)}\right], \tag{15b}$$

where:

$$A = \frac{(n-1)+(2-n)\sqrt{n(n-1)}}{\pm\mu(n-1)}, \tag{16a}$$

$$B = \frac{1}{\pm\mu}[1-\sqrt{n(n-1)}]. \tag{16b}$$

3. Numerical examinations and conclusions

For the purpose of illustrating the above considerations, the aberrational characteristics of the lens fulfilling the aplanatic condition have been determined numerically. It has been assumed that the refractive index of the glass lens $n = 1.5$, the focal length of the hybrid lens $f' = 100$ mm, while the object is at infinity. The lens parameters are, then, as follows:

$r_1 = +57.74$ mm, $r_2 = \infty$,
$R_\alpha = -48.77$ mm, $R_\beta = -45.78$,
$\varphi_R = 0.00866$, $\varphi_D = 0.00134$.

It has been assumed that $\lambda_2 = \lambda_1 = 0.0006328$ mm, relative aperture is 1:10, and tangent of the field angle does not exceed 0.05.

In Figures 2a and b, the plots of transversal spherical aberration and the field curvature are shown for the lens designed in this way. Since the deviation from the sine condition is practically zero for the whole range of the aperture angles, it has not been shown in a graph. As the spherical aberration is negligibly small, it may be concluded that the lens is aplanatic. There still exist the field curvature and astigmatism but these aberrations were not corrected.

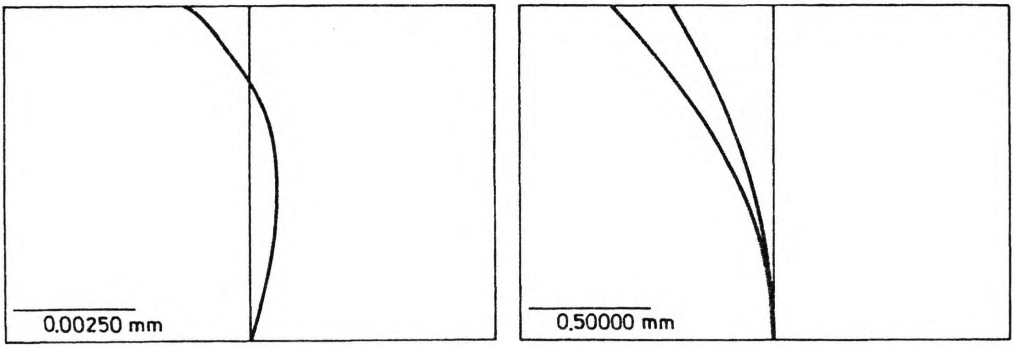

Fig. 2. Plot of the transversal spherical aberration (a), graphs of meridional and sagittal field curvature (b)

The aplanatism of the lens is even more visible when observing the shape of the aberration spot (Fig. 3). It has been determined in two ways: by ray tracing and by the method of diffraction integral calculation. For the sake of better readability, the spot diagram has been shown in the graphs, where the circle denotes the diameter of the spot containing 80% of energy, as well as the meridional cross-section of the light intensity in the aberration spot determined by diffraction method. The symmetry of the spot speaks for aplanatic correction. If the object is positioned on the axis, the lens is practically aberration free. For the field angles $y/z = 0.03$, and $y/z = 0.004$, the spot increases and becomes more diffused due to field curvature, astigmatism and high order aberrations remaining, however, still symmetric.

The same conclusions may be drawn when analysing the parameters presented in the table.

Selected parameters characterizing the aberrational spot

y/z	0	0.01	0.02	0.03	0.04	0.05
I_{max}	1.00	1.00	0.98	0.91	0.78	0.54
$D_{0.8}$ [mm]	1.2×10^{-4}	2.7×10^{-3}	6.5×10^{-3}	13×10^{-3}	22×10^{-3}	24×10^{-3}
M_2[mm^2]	2.4×10^{-7}	6.6×10^{-7}	4.5×10^{-6}	2.0×10^{-5}	5.9×10^{-5}	1.4×10^{-4}
M_3[mm^3]	0	-8×10^{-11}	-8×10^{-10}	-5×10^{-9}	-2×10^{-8}	-5×10^{-8}

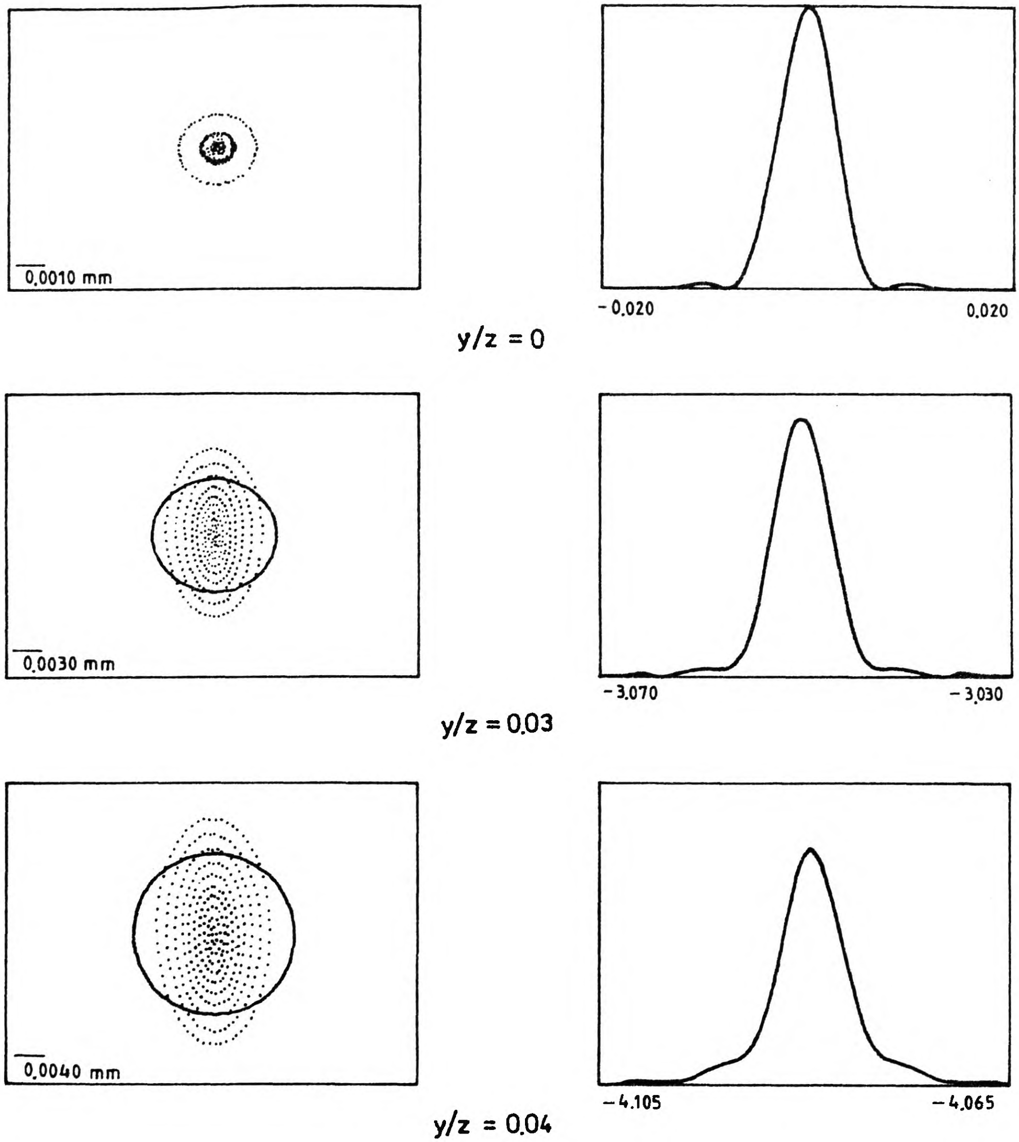

Fig. 3. Aberration spots for different field angles: a — spot diagrams, b — meridional cross-section of the intensity distribution in the spot

In the table:

I_{max} — Strehl number,

$D_{0.8}$ — spot diameter containing 80% of energy,

M_2 — second order moment of the light intensity distribution being the measure of the spot spread: $M_{2y} = \frac{1}{N}\sum_{i}^{N}(y_i-\bar{y})^2$,

M_3 – third order moment of the light intensity distribution being the measure of spot nonsymmetry: $M_{3y} = \frac{1}{N}\sum_{i}^{N}(y_i - \bar{y})^3$, $\bar{y}$ denotes the gravity centre of the spot.

The third order moment M_3 is really small for all considered values of the field angle. Additionally, it is visible that within the range of field angles up to $y/z = 0.04$ the hybrid lens fulfils the Marechal criterion. In conclusion, we may state that it is possible to design an aplanatic hybrid lens of one plane surface with a diffraction structure deposited on it. For the object positioned at infinity, the plane surface is the second refracting surface of the lens.

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