Optica Applicata, Vol. XXV, No. 4, 1995

Some comments on angle characteristic of the curved holographic optical element

E. Jagoszewski

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland.

It is well known that the characteristic function is often used in analysis and design of optical systems. This paper considers the angle characteristic of curved holographic element that has been described in [1]. If $P(x, y, z)$ is the point of intersection of the incident ray with the holographic surface, then, according to equation (7) in [1], the angle characteristic is given as

$$T = [xp_C + yq_C + (z - z_C)m_C] - [xp_I + yq_I + (z - z_I)m_I] \tag{1}$$

where (p_C, q_C, m_C) and (p_I, q_I, m_I) are direction cosines of the incident and diffracted rays, respectively. Using the same notation as in paper [1], the grating equation may be written in the form

$$\boldsymbol{n} \times (\boldsymbol{r}_I - \boldsymbol{r}_i) = 0 \tag{2}$$

where:

$$\boldsymbol{r}_I = \boldsymbol{r}_C + \frac{\lambda}{\lambda_0}(\boldsymbol{r}_O - \boldsymbol{r}_R) + \Gamma \boldsymbol{n}, \quad \Gamma = \cos\alpha_I - \cos\alpha_C - \frac{\lambda}{\lambda_0}(\cos\alpha_O - \cos\alpha_R).$$

The vector

$$\boldsymbol{r}_i = \boldsymbol{r}_C + \frac{\lambda}{\lambda_0}(\boldsymbol{r}_O - \boldsymbol{r}_R)$$

is a vector describing the direction of an actual ray diffracted at the point $P(x, y, z)$ of holographic element, whereas $\boldsymbol{r}_I$ is the unit vector of the corresponding paraxial image ray. The grating equation (2) is equivalent to the assertion that the vector $(\boldsymbol{r}_I - \boldsymbol{r}_i)$ is normal to the surface of the optical element at P and may be defined by specifying its projections on the coordinate axes: $p_I - p_i$, $q_I - q_i$, $m_I - m_i$.

The coordinates (x, y, z) of a current point of the diffracting surface may be eliminated from Eq. (1) with the help of the grating equation [2]. Calculating the partial derivatives of the speherical function

$$F(x, y, z) = z - \frac{x^2 + y^2}{2R} - \frac{(x^2 + y^2)^2}{8R^3} = 0, \tag{3}$$

we can obtain the the coordinates in the form:

$$x = -R\frac{p_I - p_i}{m_I - m_i} + \Delta x,$$

$$y = -R\frac{q_I - q_i}{m_I - m_i} + \Delta y \tag{4}$$

where Δx and Δy are quantities of the third order in p, q, x/R, y/R. The ray components are defined by the angles which the ray vector makes with the coordinate axes:

$$p = \cos\alpha, \quad q = \cos\beta, \quad m = \cos\gamma$$

where

$$m = \sqrt{1-p^2-q^2}. \tag{5}$$

To express the third coordinate in terms of the ray components, we have to substitute (4) into (3). Then we have

$$z = \frac{R[(p_I-p_i)^2+(q_I-q_i)^2]}{2(m_I-m_i)^2} + \frac{R[(p_I-p_i)^2+(q_I-q_i)^2]^2}{8(m_I-m_i)^4}$$

$$+\frac{p_I-p_i}{m_I-m_i}\Delta x + \frac{q_I-q_i}{m_I-m_i}\Delta y. \tag{6}$$

To find the expression of angle characteristic T, we substitute from (4) and (6) into (1), and the contributions involving Δx and Δy are seen to be of order higher than the third one and may be neglected. But the paraxial ray components fulfil the conditions of the object ray components, because in this case: $p_C \equiv p_R$, $q_C \equiv q_R$ and $p_I \equiv p_O$, $q_I \equiv q_O$. We then have $R = T(p_I, q_I, m_I; p_i, q_i, m_i)$ as a function of the six ray components

$$T = m_i z_i - m_O z_O - \frac{R[(p_O-p_i)^2+(q_O-q_i)^2]}{2(m_O-m_i)} + \frac{R[(p_O-p_i)^2+(q_O-q_i)^2]^2}{8(m_O-m_i)^3}. \tag{7}$$

In the above expression of the angle characteristic two of the six components may be eliminated by using Eq. (5) in expansion form

$$m_O = 1-\frac{1}{2}(p_O^2+q_O^2)-\frac{1}{8}(p_O^2+q_O^2)^2,$$

$$m_i = 1-\frac{1}{2}(p_i^2+q_i^2)-\frac{1}{8}(p_i^2+q_i^2)^2. \tag{8}$$

On substitution from (8), Eq. (7) becomes

$$T = z_i - z_O + \frac{z_O}{2}(p_O^2+q_O^2) - \frac{z_i}{2}(p_i^2+q_i^2) - \frac{R[(p_O-p_i)^2+(q_0-q_i)^2]}{(p_i^2+q_i^2)-(p_O^2+q_O^2)} +$$

$$+\frac{z_O}{8}(p_O^2+q_O^2)^2 + \frac{z_i}{8}(p_i^2+q_i^2)^2 + \frac{R[(p_O-p_i)^2+(q_0-q_i)^2]^2}{[(p_i^2+q_i^2)-(p_O^2+q_O^2)]^3}.$$

Using the notation: $u^2 = p_O^2 + q_O^2$, $v^2 = p_i^2 + q_i^2$, $w^2 = p_O p_i + q_O q_i$, we have the angle eikonal in the form

$$T = z_i - z_O + \frac{z_O}{2}u^2 - \frac{z_i}{2}v^2 - \frac{R(u^2+v^2-2w^2)}{v^2-u^2} + \frac{z_O}{8}u^4 - \frac{z_i}{8}v^4 + \frac{R(u^2+v^2-2w^2)^2}{(v^2-u^2)^3}.$$

The comments on angle characteristic of the curved holographic optical element are presented here. This observation includes another point of view of eikonal derivation, and we have the impression that it requires and further discussion.

References

[1] JAGOSZEWSKI E., Opt. Appl. **25** (1995), 71.

[2] BORN M., WOLF E., [Eds.], *Principles of Optics*, The Macmillan Co., New York 1964.

Received November 20, 1995