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Letters to the Editor

Holographic imaging of a sinusoidal test*

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The purpose of this work is to examine the influence of particular aberrations on the image contrast of an object given in the form of an amplitude sinusoidal test. As it is well known in a coherent system a transfer function for both the contrast and phase cannot be defined in a way analogical to that used in incoherent optical systems. Under certain assumptions, however, it is possible to define the degradation of contrast in the image with respect to that present in the object.

Let the amplitude transmittance of the test be defined by the formula

$$t(x_1, y_1) = a + b\cos 2\pi(\nu_x x_1 + \nu_y y_1), \quad a > b, \tag{1}$$

where ν_x, ν_y - spatial frequencies in x_1 and y_1 directions, respectively.

The light intensity may be calculated from the formula

$$I = a^2 + b^2 \cos^2 2\pi(\nu_x x_1 + \nu_y y_1) + 2ab\cos\pi(\nu_x x_1 + \nu_y y_1). \tag{2}$$

The object contrast is defined as follows:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2ab}{a^2 + b^2}. \tag{3}$$

In the paper [1] it has been shown that when the object space is assumed to be restricted so that the wave aberration be constant, the light intensity in the image plane is determined by the formula

$$I' = a^2 + b^2 \frac{P^2(x_\nu, y_\nu)}{P^2(0,0)} \cos^2\left[\frac{2\pi mR_1}{\mu R_3}(\nu_x x' + \nu_y y') + \theta\right] + 2ab\frac{P(x_\nu, y_\nu)}{P(0,0)} \times$$

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$$\times \cos\left\{k_2\left[W(0,0) - \frac{W(x_\nu, y_\nu) + W(-x_\nu, -y_\nu)}{2}\right]\right\} \cos\left[2\pi(\nu_x x' + \nu_y y') + \Theta\right], \quad (4)$$

where

$$\Theta = \frac{2\pi m R_1}{\mu}\left[\nu_x\left(\frac{x_c}{R_c} \mp \frac{\mu x_R}{m R_R}\right) + \nu_y\left(\frac{y_c}{R_c} \mp \frac{\mu y_R}{m R_R}\right)\right] + k_2\left[\frac{W(x_\nu, y_\nu) - W(-x_\nu, -y_\nu)}{2}\right], \quad (5)$$

$$x_\nu = \frac{\nu_x \lambda_2 m R_1}{\mu}, \quad y_\nu = \frac{\nu_y \lambda_2 m R_1}{\mu}, \quad (6)$$

x_R, y_R, z_R - coordinates of the reference wave source,
x_c, y_c, z_c - coordinates of the reconstructing wave source,
R_1, R_R, R_c, R' - respective distances of the object wave source, reference wave source, reconstructing wave source and image wave source,
$k_2 = 2\pi/\lambda_2$, λ_2 - wavelength of the light reconstructing the hologram,
$\mu = \lambda_2/\lambda_1$, λ_1 - wavelength of the light recording the hologram,
m - coefficient determining the hologram scale,
$W(x, y)$ - wave aberration.

Then the contrast in the image is expressed as follows

$$V' = \frac{2ab}{a^2 + b^2} \cos\left\{k_2\left[W(0,0) - \frac{W(x_\nu, y_\nu) + W(-x_\nu, -y_\nu)}{2}\right]\right\} P(x_\nu, y_\nu), \quad (7)$$

where $P(x_\nu, y_\nu)$ - pupil function defined as:

$$P(x_\nu, y_\nu) = \begin{cases} 1 & \text{in the pupil,} \\ 0 & \text{beyond the pupil.} \end{cases}$$

The drop in contrast may be expressed as follows:

$$V^* = \frac{V'}{V} \cos\left\{k_2\left[W(0,0) - \frac{W(x_\nu, y_\nu) + W(-x_\nu, -y_\nu)}{2}\right]\right\} P(x_\nu, y_\nu). \quad (8)$$

The wave aberration in the region of III order aberration is determined by the expression [2]

$$W(x, y) = -\frac{(x^2 + y^2)^2}{8} S_1 + \frac{x^3 + xy^2}{2} S_{2x} + \frac{x^2 y + y^3}{2} S_{2y} - \frac{x^2}{2} S_{3x} - \frac{y^2}{2} S_{3y} - xy S_{3xy}. \quad (9)$$

By inserting the respective terms of (9) to (8) it is possible to examine the influence of particular aberration on contrast. For the sake of simplicity, let us assume that $\nu_y = 0$, and the hologram is unrestricted, i.e., $P(x_\nu, y_\nu) = 1$. Thus, the respective drops in contrast caused by spherical aberration coma and astigmatism are

$$V^*_{sph} = \cos\left[\frac{\pi}{4\lambda_2}\left(\frac{\nu_x \lambda_2 mR_1}{\mu}\right)^4 S_1\right],$$

$$V^*_c = 1, \tag{10}$$

$$V^*_A = \cos\left[\frac{\pi}{\lambda_2}\left(\frac{\nu_x \lambda_2 mR_1}{\mu}\right)^2 S_{3x}\right].$$

It may be seen that coma does not influence the contrast degradation, but obviously it affects the imaging quality by causing a phase shift which is shown in the formula (5). However, in the case of a simple sinusoidal test used as an object this phase shift is of no practical importance.

The fact that only two aberrations decide about the imaging quality allows the reconstruction of sinusoidal object with the light of wavelength different from that of the light used for recording, provided that the hologram scale remains unchanged ($m = 1$). Thus, for instance, if $R_1 = R_R$ it suffices to fulfill the conditions [2, 3]:

$$R_c = \pm\mu R_1,$$

$$\frac{x_c}{R_c} = \pm\mu\frac{x_R}{R_R}. \tag{11}$$

Our considerations referring to an arbitrary but only single frequency are valid for a limited range, thus the conclusions should not be generalized to an arbitrary object. This is, however, a problem which - though to a lower degree - occurs always in the coherent optical imaging when the imaging quality of a complex object is evaluated by testing the image quality of a point-object.

References

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