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Resonance structure of the Mie scattering

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A simple way to define resonance in the Mie scattering and a rule to check the resonance positions are suggested.

The problem of scattering of a plane monochromatic wave by a spherical particle has been solved by Mie and Debye [1]. The dimensionless normalized scattering, extinction and absorption efficiencies, radiation pressure and specific turbidity are given by the following expressions, respectively:

$$Q_{sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2+|b_n|^2\right), \tag{1a}$$

$$Q_{ext} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\,Re(a_b+b_n), \tag{1b}$$

$$Q_{abs} = Q_{ext}-Q_{sca}, \tag{1c}$$

$$Q_{pr} = Q_{ext}-\frac{4}{x^2}\sum_{n=1}^{\infty}\frac{n(n+2)}{(n+1)}\,Re(a_n^*a_{n+1}+b_n^*b_{n+1})+\frac{2n+1}{n(n+1)}\,Re(a_n^*b_n),$$

$$\tau/c = \frac{3\pi}{\lambda x^3}\sum_{n=1}^{\infty}(2n+1)Re(a_n+b_n),$$

where a_n and b_n denote the complex partial wave amplitudes; x is the particle size parameter which equals the ratio of circumstance of the sphere to the light wavelength. Theoretical works [2, 3] on complex partial wave amplitudes and experimental studies on optical levitation [4], variation of radiation pressure [5] with wavelength of light and particle size parameter, and turbidity [6, 7] show that ripple structure in physical quantities given by eq. (1) is caused by the resonance in partial wave amplitudes a_n and b_n. For example, it has been shown [2, 3] that for dielectric spheres there is a resonance in the partial wave amplitude and in scattering efficiency if

$$x \gg 1 \text{ and } n \sim x \tag{2}$$

and

$$Re\ a_n = 1,\ Re\ b_n = 1, \tag{3a}$$

$$Im a_n = 0,\ Im b_n = 0. \tag{3b}$$

However, it has been observed that: (i) there is no unambiguous way to define resonance [2], and that (ii) the fact that $Im a_n$ or $Im b_n$ changes its sign at the centre of a resonance peak is a convenient check to find all the resonances [4]. The purpose of this paper is (i) to define resonance structure unambiguously without rereference to the resonance in partial wave amplitude, and (ii) to show that the fact that $Im\, a_n$ or $Im\, b_n$ changes its sign is not a sufficient condition to find all the resonances.

Scattering efficiency Q_{sca}, as given by eq. (1a), may also be written as

$$Q_{sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)|a_n|^2 + \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)|b_n|^2 = \sum_{n=1}^{\infty} q_e^n + \sum_{n=1}^{\infty} q_m^n = C_{em} + C_{mm},$$

where q_e^n and q_m^n represent the contribution of n-th electric and magnetic multipoles [1] and C_{em} and C_{mm} represent the contribution of all the electric and magnetic multipoles, respectively.

A program for computer ICL 2960 was written to calculate q_e^n, q_m^n, $Re\, a_n$, $Re\, b_n$, $Im\, a_n$, $Im\, b_n$, C_{em}, C_{mm} and Q_{sca}. x was varied by 0.05 steps and real refractive index was chosen to be 1.5.

Figures 1a and 1b show the variation of q_e^n and q_m^n with particle size parameter x, for $n = 1$ to 15. Fig. 2 shows the variation of C_{em}, C_{mm} and Q_{sca} with x. It is difficult to show all $Re\, a_n$, $Re\, b_n$, $Im\, a_n$ and $Im\, b_n$ curves against x, because of their large number, and full q_e^n and q_m^n curves because of their overlapping. Therefore, some important observations from these graphs have been produced in the tabular form. Tables 1 and 2 show the positions of peaks in q_e^n and q_m^n curves (the values of x for which these quantities are maximal) and the values of x, where $Re\, a_n$ and $Re\, b_n$ are unity and $Im\, a_n$ and $Im\, b_n$ are zero. From these figures and tables, the following observations can be made.

First, for a given n, both q_e^n and q_m^n curves show a number of peaks for discrete values of x such that

$$x_1^n < x_2^n < x_3^n \dots,$$

and

$$x_1'^n < x_2'^n < x_3'^n \dots,$$

where x_i^n is the position of i-th peak in q_e^n curve and $x_i'^n$ is the position of i-th peak in the q_m^n curve.

Second, since Q_{sca} is a superposition of all q_e^n and q_m^n curves we may expect all the peaks in q_e^n and q_m^n curves to show themselves in the Q_{sca} curve. However, it may be seen from Figs. 1 and 2 that not all the peaks in q_e^n and q_m^n appear in Q_{sca} curves. There are two types of peaks in q_e^n and q_m^n curves which do not do so.

First type comprises the peaks for low values of n. For example, *first peaks* of q_e^1 to q_e^8 curves corresponding to $x = 3.30$, 4.00, 4.70, 5.45, 6.15, 6.90 and 7.65 are not seen even in C_{em} curve. Similarly, first peaks in q_m^1 and q_m^2 curve corresponding to $x = 2.20$ and 2.95, though easily identifiable in C_{mm} curve, are not so in Q_{sca} curve. Thus, for real refractive index 1.5, in the Q_{sca} curve, whatever peaks up to $x = 8.000$ are only due to magnetic multipoles.

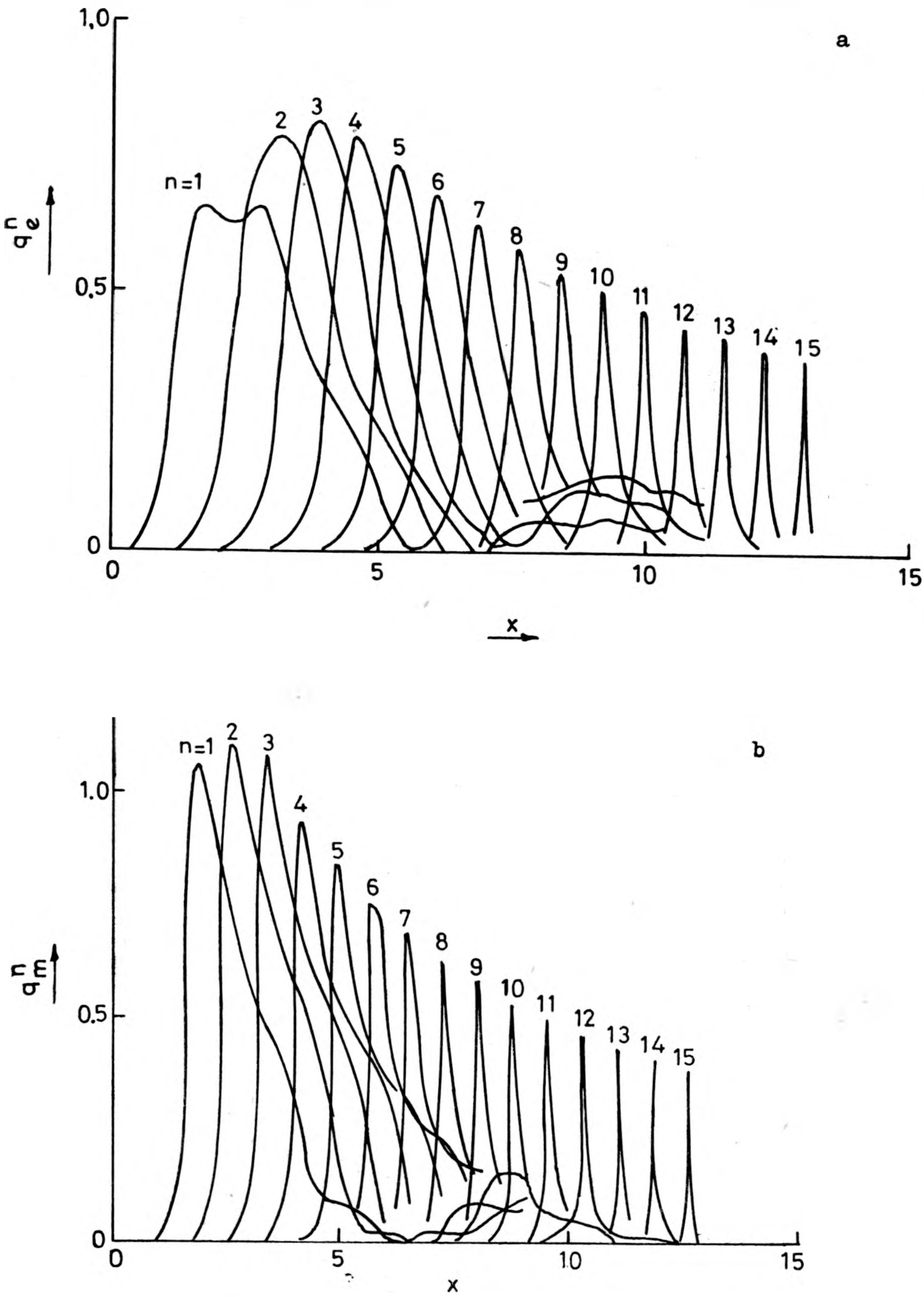


Fig. 1. a. Variation of q_e^n with particle size parameter x, refractive index – 1.5. b. Variation of q_m^n with particle size parameter x, refractive index – 1.5

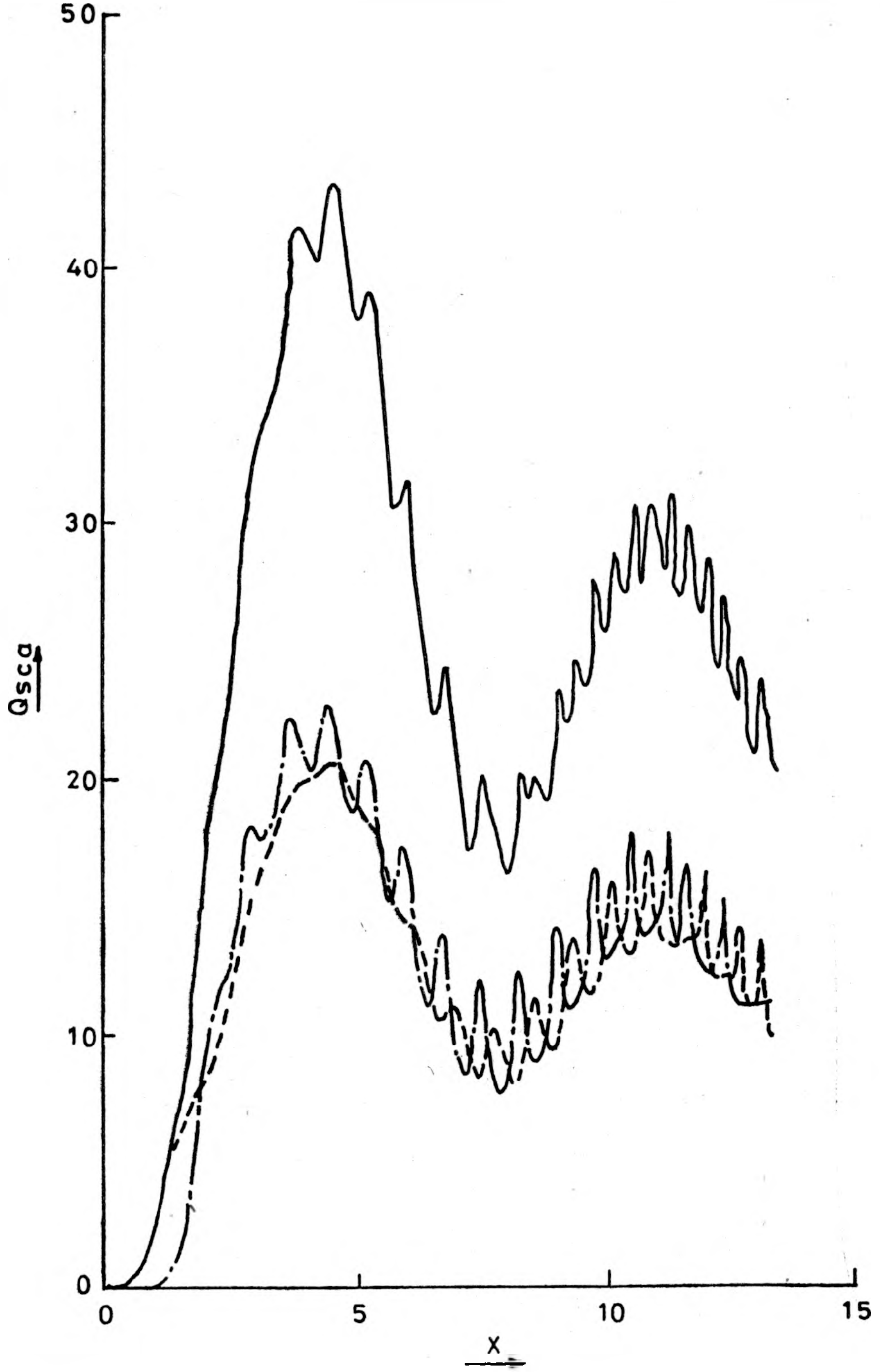


Fig. 2

Fig. 2. Variation of C_{em}, C_{mm} and Q_{sca} with particle size parameter x, refractive index – 1.5; ——— Q_{sca}, —·—·—·— C_{mm}, — — — — — C_{em}

Table 1. Positions of peaks in a_n and q_e^n curves against x

n	Value of x when $Re\ a_n = 1$ and $Im\ a_n = 0$			Value of x when a peak occurs in q_e^n curves		
	I-st position	II-nd postion	III-rd position	I-st position	II-nd position	III-rd position
1	3.15	9.40	15.70	——	——	——
2	3.75	9.95	16.20	3.30	8.75	15.10
3	4.30	9.50	15.70	4.00	9.35	15.65
4	4.85	10.05	16.30	4.70	10.00	16.25
5	5.50	10.65	16.25	5.45	9.60	15.75
6	6.25	11.00	16.35	6.15	10.25	16.30
7	6.90	11.15	16.95	6.90	10.80	16.90
8	7.65	11.55	17.45	7.65	11.40	16.55
9	8.40	12.10	17.30	8.40	12.00	17.10
10	9.20	12.70	17.70	9.20	12.55	17.65
11	9.95	13.25	18.30	9.95	13.15	18.20
12	10.70	13.85	18.85	10.70	13.75	18.80
13	11.45	14.45	19.35	11.45	14.35	19.20
14	12.20	15.05	19.80	12.20	15.05	19.55
15	12.95	15.75	20.20	12.95	15.75	20.00

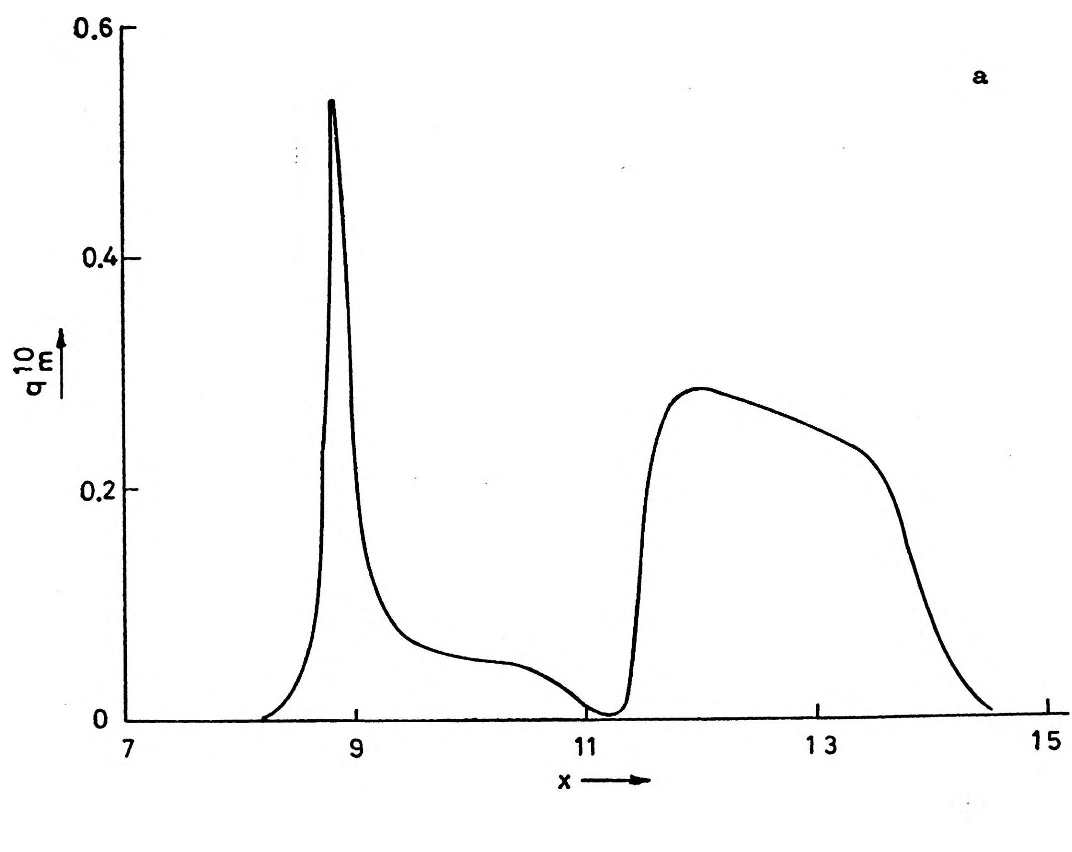
Table 2. Positions of peaks in b_n and q_m^n curves against x

n	Values of x when $Re\ b_n = 1$ and $Im\ b_n = 0$			Value of x when a peak occurs in q_m^n curves		
	I-st position	II-nd position	III-rd position	I-st position	II-nd position	III-rd position
1	3.75	9.90	16.10	2.20	8.75	15.10
2	3.25	9.45	15.70	2.95	9.35	15.65
3	3.75	10.05	16.30	3.65	8.70	15.10
4	4.45	10.50	15.75	4.40	9.35	15.65
5	5.15	10.05	16.35	5.15	10.00	16.30
6	5.90	10.70	16.95	5.85	10.65	15.75
7	6.60	11.40	16.40	6.60	11.35	16.30
8	7.35	12.05	16.95	7.35	12.00	16.95
9	8.10	12.70	17.65	8.10	11.30	17.60
10	8.85	13.05	18.30	8.85	11.95	17.00
11	9.60	12.70	18.60	9.60	12.60	17.65
12	10.35	13.30	18.30	10.35	13.30	18.25
13	11.10	14.00	18.90	11.10	14.00	18.85
14	11.85	14.70	19.56	11.85	14.70	19.60
15	12.60	15.45	20.30	12.60	15.45	

Second type includes the second and higher order peaks in q_e^n and q_m^n curves which do not appear in Q_{sca} curve. The reason why these peaks are missing in the Q_{sca} curve is that their widths (or full width at half maximum) are relatively very large, so that a superposition of such peaks contributes to smoothly varying background in the Q_{sca} curve. Having made these observations, all the peaks in q_e^n and q_m^n curve may be called as resonance peaks, irrespective of whether or not they show themselves in the Q_{sca} curve. Besides, the resonance peaks in the q_e^n or q_m^n curve corresponding to x_1^n or $x_1'^n$, respectively, may be called first order resonances. Similarly, the resonance peaks in the q_e^n or q_m^n curve corresponding to x_2^n or $x_2'^n$, respectively, may be called second order resonance. This may be generalized to define higher order resonances. If we define the resonance in this way, it is physically meaningful too. For example, x_1^3 means the lowest value of the particle size parameter, where the third electric multipole will scatter the incident radiation more strongly. Similarly, x_2^3 represents the next higher value of x for which the third electric multipole will again more strongly scatter the light.

It is interesting to note from the tables that first zero positions of $Im\, a_n$ and $Im\, b_n$ for low n do not coincide with the first resonance positions in q_e^n and q_m^n curves. This difference, however, diminishes as n decreases. This leads to the conclusion that the conditions given by eqs. (3a) and (3b) may be used to check the resonance positions only when the condition given by eq. (2) is satisfied.

Thus, for $x \gg 1$, the condition (3b) implies the change of sign of $Im\, a_n$ or $Im\, b_n$ through the centre of the resonance position. Figures 3a and 3b show the plots of q_m^{10}, and $Re\, b_{10}$ and $Im\, b_{10}$ against x. It may be seen from these figures that $Im\, b_{10} = 0$ for $x = 8.85$, 11.45 and 13.05. There is a resonance peak in the q_m^{10} curve for $x = 8.85$ and 11.95 but no resonance peak occurs for $x = 11.45$, either in the partial wave amplitude or in q_m^{10} curve. In fact, from such graphs it may be seen that between any two consecutive res-



onance positions in q_e^n or q_m^n curve there is at least one value of x for which $Im\ a_n$ or $Im\ b_n$ becomes zero and for which there is no resonance. It may be seen from the tables that the second resonance positions in partial wave amplitude and q_e^n or q_m^n curves coincide only when n and x are much larger than 1.

Thus, to check the resonance positions, the condition (3b) alone is not a sufficient condition, though it becomes the necessary one for large values of n and x. A careful examination of Figs. 3a and 3b show that as x increases, $Im\ a_n$ changes its sign at resonance positions from the positive value to the negative one. Also, this is a necessary and sufficient condition which unites both the conditions given by the eqs. (3a) and (3b). Because the only effect of a complex refractive index is to reduce the heights of the resonance peaks [2], this may be used, as a rule, both for real and complex refractive indices. Since the resonance positions in Q_{sca}, Q_{ext}, Q_{abs}, Q_{pr} and τ/c depend only on a_n and b_n, as seen from

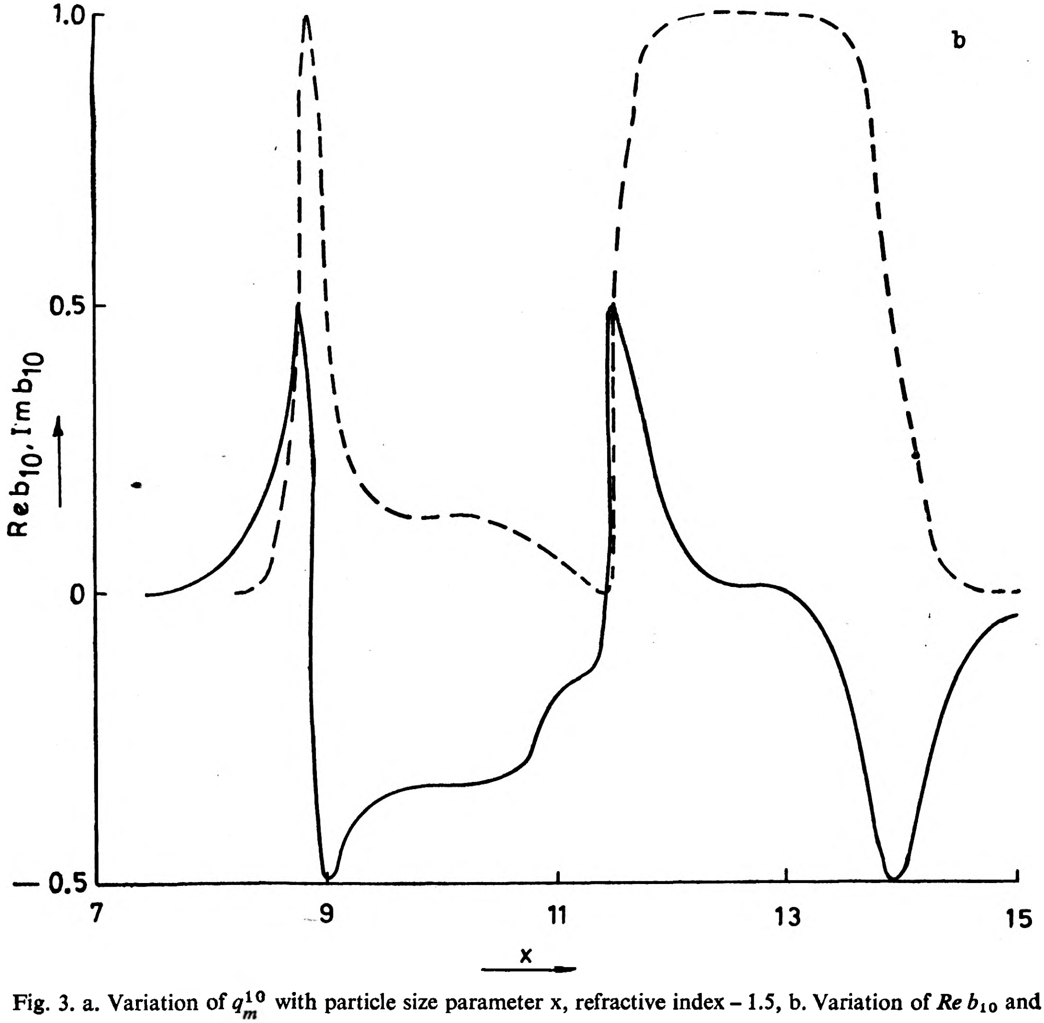


Fig. 3. a. Variation of q_m^{10} with particle size parameter x, refractive index – 1.5, b. Variation of $Re\ b_{10}$ and $Im\ b_{10}$ with particle size parameter x, refractive index – 1.5, — — — — $Re\ b_{10}$, ——— $Im\ b_{10}$

eqs. (1), the above discussion is expected to be valid for resonance structure in all these physical quantities.

This work is a part of research work on resonance structure in Mie scattering, going on in Physical Department, Indian Institute of Technology, Delhi, India.

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Резонансная структура в дисперсии Ми

Предложен простой способ определения резонансов в дисперсии Ми, а также приведено правило проверки положения резонансов.