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# Influence of film nonlinearity on the Rayleigh criterion of resolution and energy concentration

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A new approach to the analytical representation of the characteristic $(T_a - E)$ curve of holographic absorption materials by means of photometric parameters is presented. The degree of linearity of the characteristic curve is studied and experimental verification is also given. On the basis of the proposed approximation, film nonlinear influence on energy concentration in the image plane and on the image contrast in the Rayleigh criterion of resolution is considered.

## 1. Introduction

Photographic films and plates have never a perfect linear response and deviation from linearity depends to a large degree on the magnitude of the exposure variations to which the recording material is subjected. Since the amplitude transmittance $T_a$ of the material after processing is a function of the exposure $E$ and is a carrier of the information, we wish to be reconstructed, the theoretical study of the film nonlinearity leads to the analytical representation of the material $(T_a - E)$ curve. In the present paper we approximate the $(T_a - E)$ curve through some film parameters which can be estimated on the basis of the experimentally measured $(D - \lg E)$ dependence for any holographic emulsion. This approximation enables then the study of the nonlinear effects of holographic films and plates on the reconstructed wavefront. The analysis, presented here, concerns in particular the nonlinear influence of holographic amplitude materials on the Rayleigh criterion of resolution and also on the energy concentration in the zero and first orders of the diffraction pattern.

## 2. Approximation of the material $(T_a - E)$ curve

The response of the photographic layer to the exposure is conventionally described by its $(D - \lg E)$ curve, which can be measured experimentally for any chosen recording material. So, if an analytical representation of the material charac-

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teristic curve is found out, the ($T_a-E$) curve approximation is obtainable through the relationship

$$D = -2\lg T_a. \tag{1}$$

For the description of the ($D-\lg E$) curve we use the following approximation:

$$D = \frac{D_\infty}{1+\exp(a-b\lg E)+\exp(c-d\lg E)} \tag{2}$$

where $D_\infty$ is the maximum of the measured optical density $D$, and $a, b, c, d$ are parameters which can be determined from the experimentally obtained ($D-\lg E$) curve. Equation (2) may be considered as consisting of two terms The first term is

$$D = \frac{D_\infty}{1+\exp(a-b\lg E)}, \tag{3}$$

and resembles the Fermi formula of electron energy distribution. Curve 2 in Fig. 1 represents the diagram of the approximating function (3), curve 1 is the diagram of function (2) which approximates the experimentally measured $D-\lg E$) curve of the photographic material, and the matching fit accuracy s $\Delta D \cong 0.01$. As we can see, both of the curves, except for their periods of

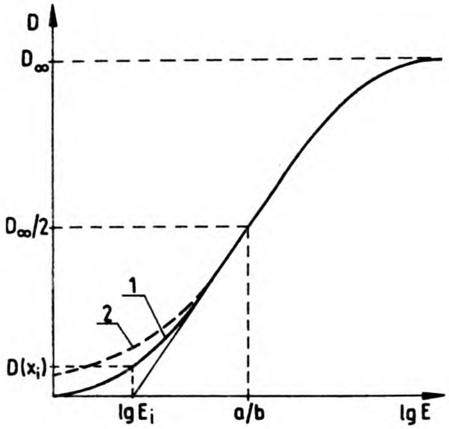

Fig. 1. The $D-\lg E$ curve of a hypothetical recording material curve 1 – approximation of the ($D-\lg E$) curve by formula (2) curve 2 – approximation of the ($D-\lg E$) curve by formula (3)

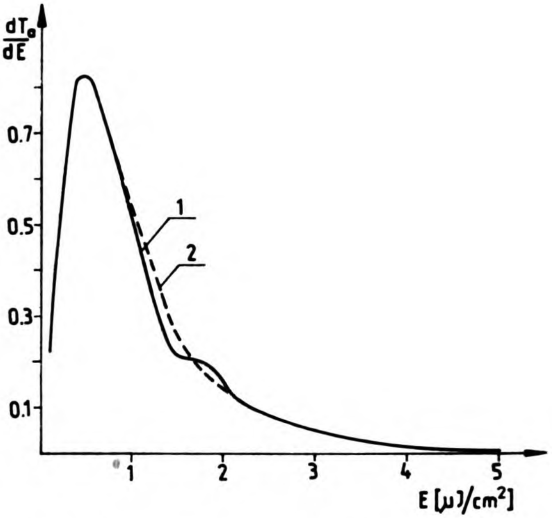

Fig. 2. Exposure derivative of the experimental (1) and the approximated (2) amplitude transmittance $T_a$ for the Agfa Gevaert 10E56 emulsion. Developing conditions: developer G280C, time 4 min., temperature 20°C

under-exposure, are similar: they have almost the same slope $\gamma$ and the linear parts of the curves overlap. The behaviour of curves 1 and 2 for exposures $E > E_i$, where $E_i$ is the exposure at the inertial point, is determined by the values of the parameters a and b. The latter can be estimated from the experimental $(D - \lg E)$ curve by the following expressions:

$$b = \frac{4\gamma}{D_\infty}, \tag{4}$$

$$\frac{D_\infty}{2} = \gamma \cdot \left(\frac{a}{b} - \lg E_i\right)$$

where the numerical values of $D_\infty$, $E_i$ and $\gamma$ are obtained from the experimental $(D - \lg E)$ curve. The parameters c and d in the second exponential term $\exp(c - d\lg E)$ in Eq. (2), which we call an under-exposure correction, are obtainable from the following equations:

$$c = \ln \frac{\left(D_\infty/D(x_i/2) - \{1+\exp[2(1+x_i/D)]\}\right)^2}{D_\infty/D(x_i) - (1+e^2)}, \tag{5}$$

$$d = \frac{c - \ln[D_\infty/D(x_i) - (1+e^2)]}{x_i}$$

where $x_i = \lg E_i$ and $D_\infty$, $E_i$, $\gamma$, $D(x_i)$, $D(x_i/2)$ are estimated from the experimental $(D - \lg E)$ curve. The relations expressed by means of Eqs. (4) and (5) can be derived from Eqs. (2), (3) and from the analytical formula of the linear part with a slope $\gamma$ of curves 1 and 2 (Fig. 1) at the point $\lg E = a/b$.

Substituting Eq. (2) into Eq. (1), we find the respective analytical representation of the $(T_a - E)$ curve

$$T_a = 10^{-D/2} = 10^{-D_\infty/2[1+\exp(a-b\lg E)+\exp(c-d\lg E)]}. \tag{6}$$

Differentiation of Eq. (6) yields the degree of linearity of the $(T_a - E)$ curve. Figure 2 represents the dependence of the exposure derivative of $T_a$ on the magnitude of $E$ for the experimental curve (1) and the approximated curve (2) of the Agfa Gevaert 10E56 emulsion. It turns out that the linear parts of both curves coincide and that they occur in a rather limited region, namely in the vicinity of $E \simeq 0.56$ μJ/cm², which corresponds to the value of the amplitude transmittance $T_a \simeq 0.72$.

## 3. The film nonlinear influence on the Rayleigh criterion of resolution

According to the so-called Rayleigh condition of resolution two incoherent point sources are resolved, if the maximum of the irradiance distribution of one of the sources overlaps the minimum of the irradiance distribution generated by the second source. The minimum resolvable separation of the geometric images is then

$$2b' = \frac{3.832\,q}{kl} \tag{7}$$

where $l$ is the semidiameter of the circular exit pupil, $k$ is the wave number and $q$ is the image distance in a diffraction-limited system.

It will be interesting now to estimate whether these two point sources would be easier or harder to resolve if the image irradiance distribution has been recorded in a photographic emulsion. Let us consider a simple two-dimensional optical system that consists of a single lens (see Fig. 3), assuming that the lens is diffraction-limited. According to the scalar diffraction theory, the intensity distribution in the image plane under the Rayleigh condition of two-point resolution Eq. (7) is expressed in the following form:

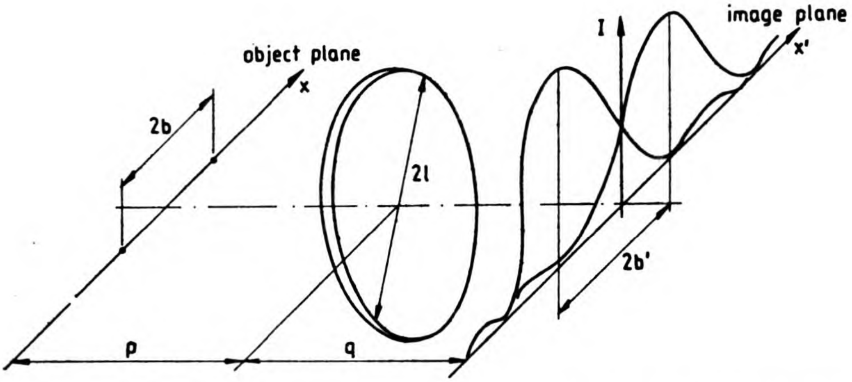

Fig. 3. Geometry for image formation in a diffraction-limited system consisting of a single lens

$$I(z) = K\left\{\left[\frac{2J_1(z-1.916)}{z-1.916}\right]^2 + \left[\frac{2J_1(z+1.916)}{z+1.916}\right]^2\right\} \tag{8}$$

where $z = \frac{klx'}{q}$, $J_1$ is a Bessel function of the first-kind, first order and $K$ is a constant factor.

We define the image contrast $C$ as the ratio

$$C = \frac{I_{\max}(z) - I_{\min}(z)}{I_{\max}(z) + I_{\min}(z)} \tag{9}$$

where $I_{\max}$ is the maximum value of the intensity distribution in the image plane, while $I_{\min}$ is the minimum. The evaluation of Eq. (9) gives $C_t = 0.153$ which is the value of the theoretical image contrast of two incoherent point sources separated by the Rayleigh distance.

Let us consider now the film nonlinear influence on the image contrast in the Rayleigh criterion. A square-law recording material (e.g., a photoemulsion) responds to the irradiance distribution $I(z)$ and the exposure $E$ is

$$E(z) = tI(z) \tag{10}$$

where $t$ is the exposure time. From Fig. 4 we can infer that the contrast in the image plane will be affected mainly by the location of the magnitudes $I_{\max}$ and $I_{\min}$ on the $(T_a - E)$ curve. The degree of linearity of the characteristic curve at these locations will be then responsible for a decrease or an increase of the image contrast. Therefore, it is important to know which part of the film characteristic curve should be applied when the highest image contrast is demanded. Through the analogy of Eq. (9), we define the image contrast of the intensity distribution recorded in a photographic layer by the ratio

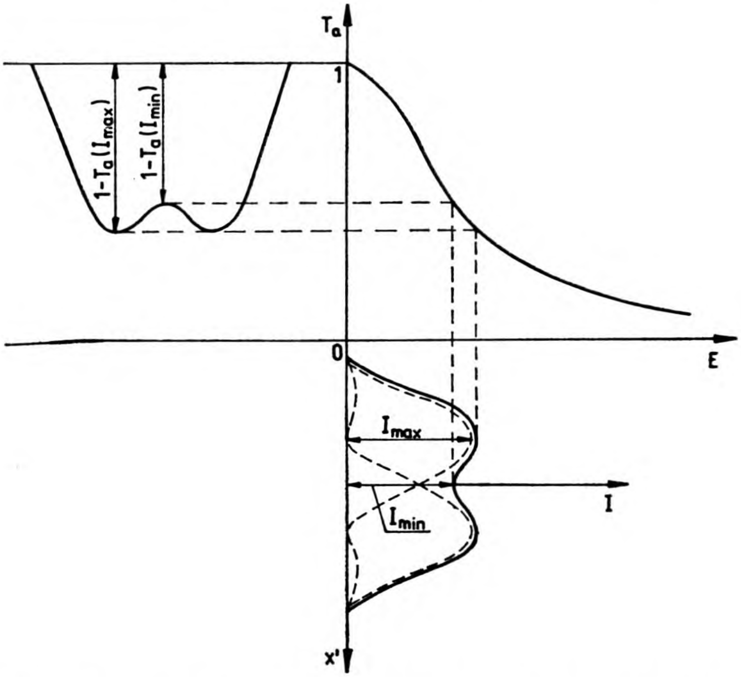

Fig. 4. The nonlinear influence of the $(T'_a - E)$ curve of a hypothetical recording material on the intensity distribution in the image plane

$$C_p = \frac{T_a^2(I_{\min}) - T_a^2(I_{\max})}{2 - T_a^2(I_{\max}) - T_a^2(I_{\min})}. \tag{11}$$

Substituting Eq. (8) into (10) and then in Eq. (6), we can calculate the image contrast of the irradiance distribution recorded in a particular photographic emulsion. Our experiments and calculations refer to Agfa Gevaert's 10E56 emulsion and the numerical results are plotted in Fig. 5. As we can see, maximum image contrast is attainable at $E = 0.56\ \mu J/cm^2$ which corresponds to the value of the amplitude transmittance $T_a = 0.72$. In literature treating nonlinearities of recording media there is an ambiguity in reference to the metric units of the applied exposure. To allow a direct intercomparison of various amplitude transmittance curves, we have marked in Fig. 5 the values of $T_a$ which correspond to the given numerical values of the applied exposure. So, image contrast $C_p$ greater than the theoretical contrast $C_t$ is obtainable in the region of $T_a = 0.43$–1 of the ($T_a - E$) curve. Over-exposure of the photographic emulsion results in a sharp decrease of the image contrast.

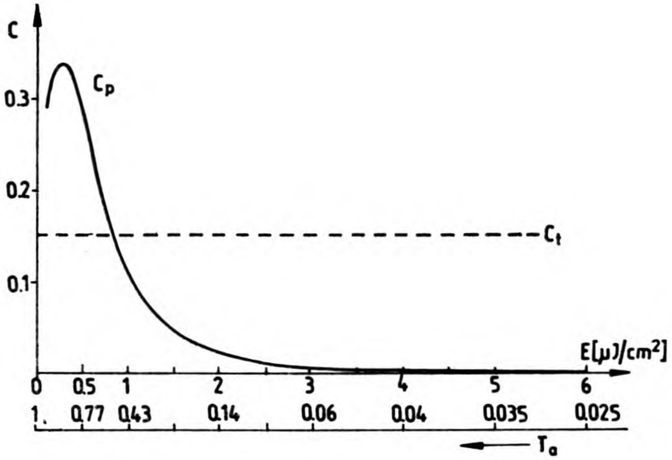

Fig. 5. Image contrast of two-point sources separated by Rayleigh distance if the intensity distribution is recorded in Agfa Gevaert's 10E56

## 4. The film nonlinear influence on energy concentration

We analyse this problem through the example of a single lens which is assumed to be diffraction-limited. The impulse response of such a system obtained with coherent illumination is

$$I(r) = K\left[\frac{2J_1\left(\frac{klr}{f}\right)}{\frac{klr}{f}}\right]^2 \tag{12}$$

where $K$ is a constant factor and $f$ is the lens focal length.

The total energy concentration in the diffraction pattern (Airy's pattern) with radius $c$ equals

$$\mathcal{E} = \iint\limits_S I(r)\,dS = \int\limits_0^c\int\limits_0^{2\pi} I(r)\,r\,dr\,d\varphi = 2\pi\int\limits_0^c I(r)\,r\,dr \tag{13}$$

where S is the integral surface.

If the intensity distribution expressed by Eq. (12) is recorded in a particular holographic emulsion, the total energy of the diffraction pattern with radius $c$ will be

$$\mathcal{E} = 2\pi\int\limits_0^c T_a^2(It)\,r\,dr \tag{14}$$

where $T_a$ is the amplitude transmittance of the recording material and $t$ is the exposure time. Numerical calculations of Eq. (14), on the basis of the approximating formula (6), reveal then the film nonlinear effects on energy concentration for different values of the limit of integration $c$. Our results for Agfa Gevaert's 10E56 holographic emulsion are plotted in Fig. 6. We have found that an appropriate quantitative valuation of energy concentration is done in terms of the ratio $\mathcal{E}_1/\mathcal{E}_0$, where $\mathcal{E}_1$ is the first-order energy, while $\mathcal{E}_0$ is the zero-order energy of the diffraction pattern. Figure 6 reveals that energy concentration is greatly influenced by the magnitude of the exposure to which the recording medium is subjected. Low exposures give higher values of the ratio $\mathcal{E}_1/\mathcal{E}_0$. Over-exposures result also in a slight increase of the ratio $\mathcal{E}_1/\mathcal{E}_0$, however they refer to the toe part of the $(T_a - E)$ curve, which is rarely used in practice because of its low image contrast and diffraction efficiency [3]. The minimum of ratio $\mathcal{E}_1/\mathcal{E}_0$ occurs at the point of $E \cong 2.3$ μJ/cm² which

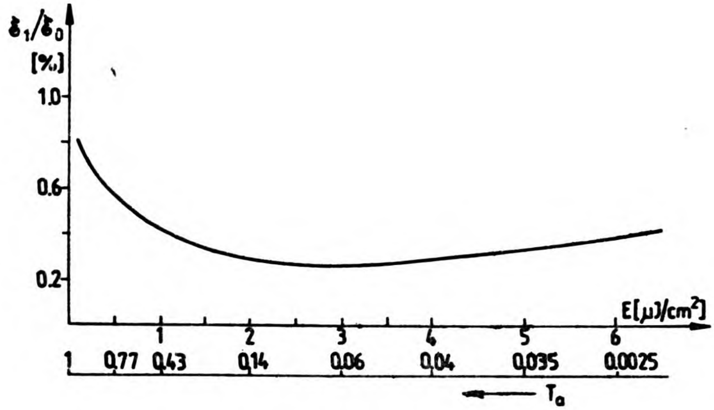

Fig. 6. $\mathcal{E}_1/\mathcal{E}_0$ as a function of the applied exposure to Agfa Gevaert's 10E56

corresponds to $T_a \cong 0.1$ and is in the vicinity of the inflexion point of the $(T_a - E)$ curve.

## 5. Concluding remarks

The presented analysis and its results refer to the Agfa Gevaert 10E56 emulsion. However, we have successfully applied the analytical representation through Eq. (6) to the Kodak 614F emulsion and the Bulgarian HP-490 and HP-650 holographic plates. The proposed approximation involves photometric parameters which can be determined for any absorption holographic recording material. We find this method to be much more convenient for adaptation than the commonly used polynomial approximations of the $(T_a - E)$ curve which utilize complex numerical calculations [1, 2].

Our analysis yields high nonlinearity of the $(T_a - E)$ curve. The linear part of a practical holographic emulsion occurs in a limited region and its correct estimation optimizes image contrast. Through the example of two-point sources separated by the Rayleigh distance, we have shown a considerable decrease of the image contrast, if a nonlinear part of the film characteristic $(T_a - E)$ curve is applied. However, the degree of linearity does not seem to be a decisive factor if energy concentration is considered. Operation beyond the linear part of the characteristic curve may result in higher or lower values of energy concentration than that corresponding to the linear part.

The presented analytical approach of the film characteristic curve may be used also in the study of film nonlinear influence on hologram diffraction efficiency [3], harmonic and intermodulation noise, or some other useful characteristics in holographic image formation. This, however, is beyond the considerations taken up in this paper.

## References

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### Влияние нелинейности эмульсии на критерий разрешения Рэлея и концентрацию энергии

Представлен новый подход к аналитическому изображению характеристической кривой $(T_a - E)$ голографических абсорбционных материалов при помощи фотометрических параметров. Исследована степень нелинейности характеристики и проведена экспериментальная проверка. На основе предложенной аппроксимации рассмотрено влияние нелинейности эмульсии на концентрацию и контрастность Рэлея в плоскости изображения.