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Admittance loci in direct level monitoring of nonquarterwave multilayer filters*

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Analytical formulae are described for δ-scale of the admittance locus of the homogeneous and nonabsorbing layer of arbitrary thickness deposited on any multilayer. Application of admittance loci of nonquarterwave multilayer filter to direct level monitoring of the filter at a chosen wavelength is also presented.

1. Introduction

In 1972 Macleod [1] published the admittance diagram approach to study errors and mechanism of error compensation in direct turning value monitoring of quarterwave narrow-band interference filters. The same techniques were used for consideration of dynamic errors [2] and investigation of the monitoring of quarterwave antireflection coatings and edge filters [3, 4].

Formulae given in [1] can be applied only in construction of the admittance locus of homogeneous and nonabsorbing layers of quarterwave optical thickness. To employ the admittance loci technique for a purpose of direct level monitoring of filters containing layers of arbitrary thicknesses it is necessary to have formulae for δ-scale of the admittance locus valued in the case when locus of layers starts and ends at any point of complex plane. Description of these formulae and their use in optical monitoring are the aim of this contribution.

2. Analytical calculations

Real and imaginary parts of the optical admittance $Y = a+ib$ of homogeneous and nonabsorbing layer of index N and phase thickness δ may be expressed by two following equations [1]:

$$a[\cos\delta-(\beta/N)\sin\delta]-(ba/N)\sin\delta = a\cos\delta, \tag{1}$$

$$b[\cos\delta-(\beta/N)\sin\delta]+(aa/N)\sin\delta = N\sin\delta+\beta\cos\delta \tag{2}$$

where $a+i\beta$ is the complex admittance of the substrate.

It has been shown [1] that the locus of the admittance can be represented on its complex plane (Im Y, Re Y) by the circle with the centre on real axis of Y.

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Further, in the simplest case, where phase thicknesses of layers are $\delta = \pi/2, \pi, \ldots,$ the locus of Y starts and ends also on the real axis of Y.

For the present purpose of the analysis it is necessary to derive formulae for the coordinates (a, b) of the point on the locus of Y that represents the layer of any given phase thickness δ. It should be assumed, moreover, that the layer is deposited on an arbitrary multilayer, represented by the complex vaue of admittance $\alpha + i\beta$. These formulae can be obtained by solving Eqs. (1) and (2) with respect to a and b. Introducing the auxiliary quantities

$$\eta = N \sin\delta + \beta \cos\delta \tag{3}$$

and

$$\varkappa = \cos\delta - (\beta/N)\sin\delta \tag{4}$$

it can be found that

$$b = \frac{N\varkappa\eta - \alpha^2 \cos\delta \sin\delta}{N\varkappa^2 + (\alpha^2/N)\sin^2\delta} \tag{5}$$

and

$$a = [\alpha\cos\delta + (b\alpha/N)\sin\delta]/\varkappa. \tag{6}$$

Inserting Equations (3) and (4) into Eq. (5) and performing all the necessary calculations, we can derive the following formula for b:

$$b = N\frac{(N^2 - \alpha^2 - \beta^2)\tan\delta + \beta N(1 - \tan^2\delta)}{(N - \beta\tan\delta)^2 + (\alpha\tan\delta)^2}. \tag{7}$$

In the similar way the formula for a can be found

$$a = \frac{\alpha N}{N - \beta\tan\delta} \times \frac{N^2 - N\beta\tan\delta + N^2\tan^2\delta - \beta N\tan^3\delta}{(N - \beta\tan\delta)^2 + (\alpha\tan\delta)^2}. \tag{8}$$

3. Admittance loci of filter's layers and its monitoring

Let us consider, as a simple example, a cemented three-layer nonquarterwave filter for increasing colour temperature of the light source. Designs of such filters were published in paper [5]. In view of the fact that the structure of layer depends on the way of its preparation, which is different in different laboratories, it is necessary to look for the design solutions more closely fitted to the dispersion data of real layers [6].

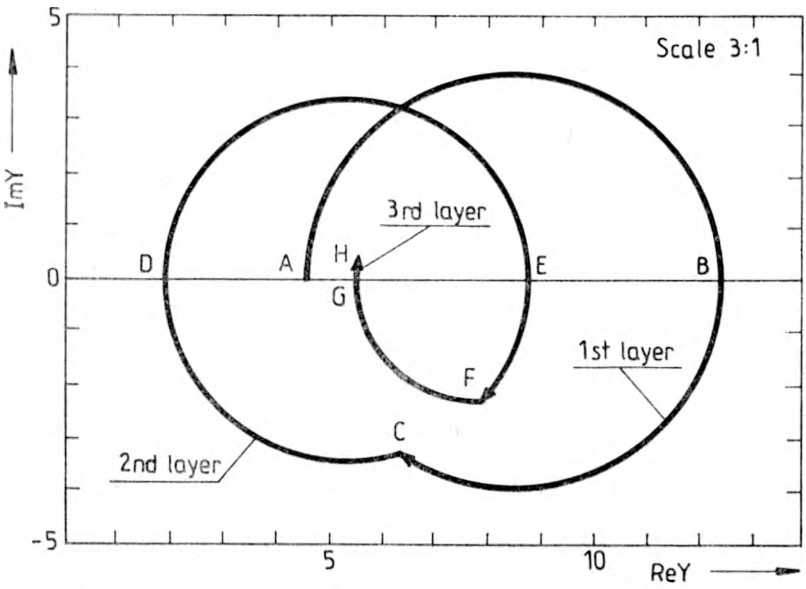
The design of the filter, obtained by synthesis is shown in Table 1. For further simplicity let us assume that the optical monitoring of the whole filter is carried out using monochromatic light of a single wavelength only. While analysing the admittance locus of the filter's subsequent layer the value transmittance corresponding to the point (a, b) on the complex plane of admittance is calculated from the formula

$$T = 4a/[(1+a)^2+b^2]. \tag{9}$$

Table 1. Design of nonquarterwave three-layer filter

Layer number	Material	Index at 425 nm	Layer thickness [nm]
Substrate	Glass	1.515	
1	Zinc sulphide	2.4956	69.2
2	Cryolith	1.33	162.6
3	Zinc sulphide	2.4956	29.4
Medium	Optical cement	1.489	

Admittance locus of the first layer, made of zinc sulphide, starts at the point A (see Figure), then it crosses the real axis at the point B. This point corresponds to the turning value of trasmittance, the deposition is, however, not stopped but continues till the layer reaches the transmittance corresponding to the point C, i.e., the final point for the first layer. The coordinates of this point may be calculated from the formulae (5) and (6) or (7) and (8). The value of transmittance calculated at this point may be used as the test value for the real deposited layer. The same meanings have the other test points D and G, but in the case of a direct monitoring they should be regarded as the test points for the existing multilayer. The locus of admittance of the following layer, made of cryolith, starts at the point C then it crosses two test points D and E and terminates at F. The admittance locus of the last layer starts at the point F, it crosses test point G



Admittance loci of nonquarterwave three-layer filter at 452 nm and reaches the final point H. All the points and the corresponding values of transmittance are listed in Table 2.

Table 2. Values (levels) of transmittance considered during direct single-wavelength level monitoring of the filter

Point	Value (level) of transmittance	Interpretation of point
A	0.958	Initial for the first layer
B	0.629	Test
C	0.774	Final for the first layer Initial for the second one
D	0.939	Test
E	0.759	Test
F	0.764	Final for the second layer Initial for the third one
G	0.916	Test
H	0.913	Final for the third layer

4. Conclusions

The paper presents the analytical formulae for determining the coordinates of the point that reaches the optical admittance locus of homogeneous and nonabsorbing layer having arbitrary phase thickness. The layer is considered to be deposited on any multilayer. The admittance loci of nonquarterwave multilayer were used for predicting both termination and test values of transmittances during the direct level monitoring of the filter at the given wavelength.

References

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Диаграммы полной оптической проводимости для непосредственной проверки уровня многослойных фильтров с нечетвевртьволновыми оптическими толщинами

Аналитические формулы описывают щкалу диаграммы полной проводимости однородного непоглощающего слоя любой толщины, нанесенного на любой многослой. Представлено, кроме того, применение диаграмм полной проводимости многослойных фильтров с нечетвертьволновыми оптическими толщинами для непосредственной проверки уровня проводимости фильтра для избранной длины волны.