Optica Applicata VII/3, 1977

Florian Ratajczyk *

Influence of bubbles on the background distribution due to scattering in the optical image

A first attempt has been made to evaluate approximately the illumination intensity in the image plane of the optical instruments caused by the light scattered by the bubbles. It is hoped, that the results of this work may be a basis for a rational tolerancing of the permissible bubble content in the optical elements.

1. An indicatrix of scattering by a bubble in glass

The bubbles belong to a small group of glass discontinuity. It is evident that the bubbles occurring in the glass elements of the optical instruments may influence the imaging quality. The deterioration of imaging quality by bubbles has not been discussed in the literature so far and the present paper is an attempt to describe certain aspects of this problem. A bubble in an optical system is essentially a scattering element. G. E. Davis considered a theoretical indicatrix of light scattering by a bubble immersed in water (fig. 1) [1]. This curve is similar in the general shape to the experimental curve of relative directional luminous intensity obtained by us for the bubbles in the borosilicate glass (fig. 2).

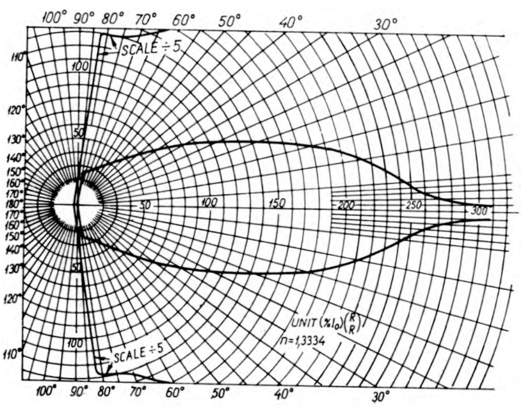

Fig. 1. Indicatrix of directional luminous intensity of a bubble immersed in water, according to G. E. Davis [1]

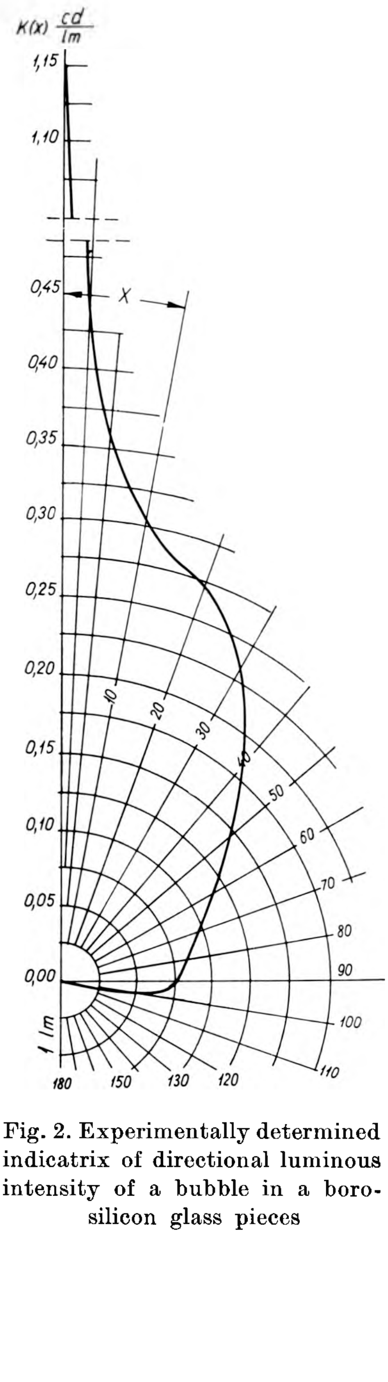

Fig. 2. Experimentally determined indicatrix of directional luminous intensity of a bubble in a borosilicon glass pieces

* Institute of Physics, Wrocław Technical University, Wrocław, Poland.

The relative directional luminous intensity $K(X)$ is defined as the ratio of the luminous intensity propagating under the angle X to the direction of the incident light beam to the flux of this beams.

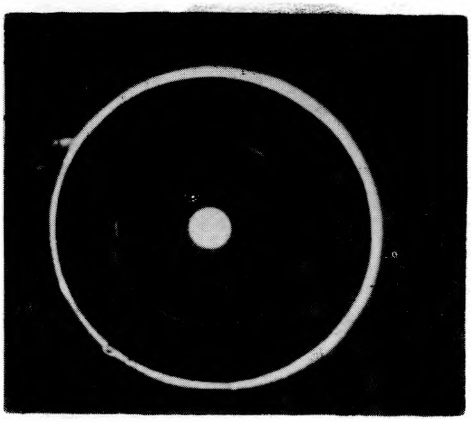

Fig. 3. A view of a bubble within glass when observed against the incident light

The shape of the directional curve of the luminous intensity for a bubble remains unchanged, if the covergence point of the beam illuminating the bubble is distant from the latter by no less than 5 diameters of the bubble.

The following simplified physical model of the bubble may be assumed which is useful for estimation of its influence on the imaging quality.

The bubble in an optical system may be considered as a nontransparent target with an infinitesimally small shining element placed in its middle and possessing a relative directional luminous intensity indicatrix as plotted in fig. 2.

The correctness of such an approach is confirmed by a picture of a bubble observed in the direction opposite the incident light, presented in fig. 3.

2. An imaging of an off-axial object element by the optical system with a bubble located on its optical axis

The case defined in the title of this section is shown schematically in fig. 4. A surface element ΔS of luminance B characterized by the Lamberts law is positioned at a distance s from the object principal plane of the optical system. A bubble of cross-section area equal to σ_p is located on the optical axis at the distance l from the system. The image of the bubble appears at a distance l'. This situation results in a shining background which occurs at the plane of the true image plane (due to spread image of the bubble at this plane). The relative distribution of this shining background is to be evaluated.

For this purpose we choose a point of coordinates (r, φ) in the principal plane. The straight line connecting this point with the bubble creates an angle α with the optical

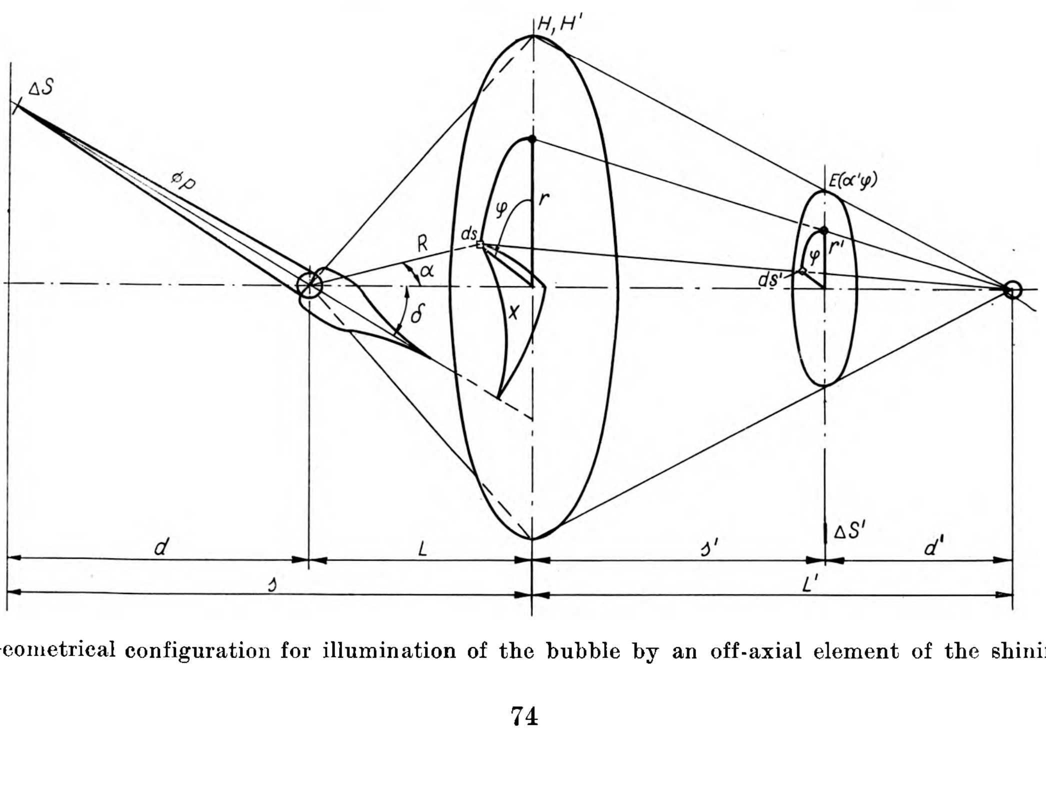

Fig. 4. Geometrical configuration for illumination of the bubble by an off-axial element of the shining surface

axis and an angle X with the indicatrix axis. The angle X may be determined on the base of spherical trigonometry by a formula

$$X = \arc\cos(\cos\alpha\cos\delta - \sin\alpha\sin\delta\cos\varphi). \quad (1)$$

The luminous intensity of the bubble in the X direction is determined by

$$I(X) = K(X)\Phi_p,$$

where Φ_p denotes the light flux falling into the bubble and is equal to

$$\Phi_p = B\Delta S\, W_p \cos\delta = B\Delta S \frac{\sigma_p}{d^2}\cos^3\delta,$$

where W_p denotes a solid angle subtended by the bubble from the middle of the object surface element ΔS, d denotes the distance between the bubble and the object plane.

By respective substitution we get

$$I(X) = K(X) B\Delta S \frac{\sigma_p}{d^2}\cos^3\delta. \quad (2)$$

The bubble shining toward the point (r, φ) causes an illumination of the principle plane

$$E(r, \varphi) = \frac{I(X)}{R^2}\cos\alpha = \frac{I(X)\cos^3\alpha}{l^2}. \quad (3)$$

The light flux passing through a surface element dS in principal plane illuminates an element dS' of correspondingly less surface in the image plane.

The ratio of the intensity of illumination E' of the element dS' in the image plane to the intensity E in the principal plane is equal to r^2/r'^2, in other words

$$E(r', \varphi) = E(r, \varphi)\frac{l'^2}{d'^2}\tau,$$

where τ is a transmission coefficient of the optical system.

After substitution of (2) and (3) we obtain

$$E(r', \varphi) = K(X) B\Delta S\, \sigma_p \cos^3\delta \cos^3\alpha \frac{\beta_p^2}{d'^2 d^2}\tau, \quad (4)$$

where $\beta_p = l'/l$ is the transversal magnification of the bubble.

The relation between the coordinates of the point (r', φ) and the corresponding directional angle X of the indicatrix may be easily found. As it is easily seen from fig. 4 the coordinate r' determines the angle α in accordance with the formula

$$\alpha = \arc\tan\left|\frac{r'\, l'}{d'\, l}\right|. \quad (5)$$

The angle δ is defined by the position of the object element ΔS. With the help of (1) it is possible to assign the respective angle X to each angle φ.

The intensity of illumination in the image produced by an optical instrument is determined by a well known formula [2]:

$$E' = \pi\tau B\sin^2 u', \quad (6)$$

where u' is an image aperture angle.

If there is a bubble in the light beam the corresponding drop of illumination intensity in the geometrical image (shadow) amounts to:

$$E' = \pi\tau B\left(1 - \frac{\sigma_p}{\sigma}\right)\sin^2 u', \quad (7)$$

where σ is the cross-section area of aperture beam in the plane of the bubble.

Thus, in the image plane an image of intensity E' occurs together with an illuminating background of illumination intensity $E(r', \varphi)$ determined by the formula (4).

By adding both the intensities and dividing them by the intensity E_0, which would occur if there were no bubbles, we get the formula for the relative distribution of the illumination intensity in the image plane

$$E_w = \underbrace{1}_{a} - \underbrace{\frac{\sigma_p}{\sigma}}_{b} + \underbrace{\frac{K(X)\,\Delta S\,\sigma_p \cos^3\delta\cos^3\alpha}{\pi\sin^2 u'}\,\frac{\beta_p^2}{d'^2 d^2}}_{c}. \quad (8)$$

In the formula (8) 3 parts may be distinguished:

Part a) represents a relative intensity of illumination in the elements ΔS of the image produced by an optical system without any bubble. Part b) represents the action of the shadow produced by the bubble, while part c) — is the shining background due to light scattered by the bubble.

For the finite surface S we get after integrating

$$E_w = 1 - \frac{\sigma_p}{\sigma} + \frac{\sigma_p \cos^3\alpha\beta_p^2}{\pi\sin^2 u' d'^2 d^2}\iint_S K(X)\cos^3\delta\, dS. \quad (9)$$

A measure of harmfulness of the shining background in a given optical system is the relative illumination intensity at the brightest point of the background for the most disadvantageous location of both the bubble and the object surface S. This occurs, when the bubble is positioned on the optical axis, and the surface S is a circle with the centre located also on the optical axis, if simoultaneously $\alpha = 0$ which results on $X = \delta$ from (1). Then we have

$$E_{w\max} = 1 - \frac{\sigma_p}{\sigma} + \frac{\overline{K}(\delta) S \sigma_p}{\pi \sin^2 u'} \frac{\beta_p^2}{d'^2 d^2}, \tag{10}$$

Table 1

Values of $\overline{K}(\delta)$

Range of δ	$\overline{K}(\delta)$
$(-5°, 5°)$	0.6
$(-10°, 10°)$	0.4
$(-15°, 15°)$	0.36
$(-20°, 20°)$	0.33

where $\overline{K}(\delta)$ denotes the value $\langle\langle K(\delta) \cos^3 \delta \rangle\rangle$ averaged in the region of field angle of the surface S, if seen from the bubble position point.

By introducing the notation:

$$\sigma_w = \frac{\sigma_p}{\sigma}; \qquad \Omega = \frac{\sigma}{d^2};$$

$$\omega = \frac{S}{d^2};$$

and using the Newtons coordinates (fig. 5) we get the following formula for one bubble

$$E_{w\max} = 1 - \sigma_w + \frac{\overline{K}(\delta) \Omega \omega \sigma_w}{\pi \sin^2 u'} \frac{x^2}{f^2}. \tag{11}$$

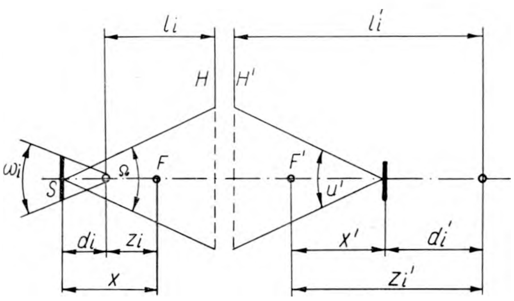

Fig. 5. Geometrical configuration in the case of illuminating the bubble with a spread surface S

If in the vicinity of the optical axis there are N bubbles, then, as it is clear from fig. 6, the brightest points in the background produced by each bubble are summed up giving a maximal value of the all-over relative intensity:

$$E_{w\max} = 1 - \sum \sigma_{w_i} + \frac{\Omega \sum_{i=1}^{N} \overline{K}_i(\delta) \sigma_{wi} \omega_i}{\pi \sin^2 u'} \frac{x^2}{f^2}. \tag{12}$$

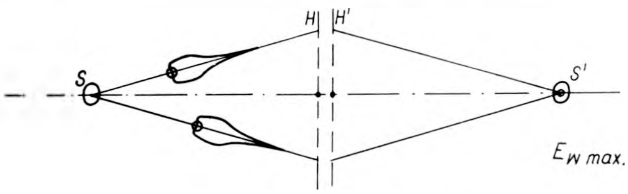

Fig. 6. N bubbles occurring in an optical system; their maxima of background illumination intensities are summed up in the image plane

The formula (12) may be used to evaluate the drop in the relative illumination intensity produced by the bubble shadows, and the maximal relative intensity of the light scattered in the image plane.

3. An experimental verification of the theoretical influence of the bubble on the quality of the geometrical imaging for an off-axial element of the object

The case of an off-axial object element is a particularly suitable for the observation, because it allows to separate the shining background region from the surface element image. The verification was made by quantitative and qualitative checking of the conclusions following from the formula (8).

The measurement were carried out in a model optical system aligned along an optical bench. The scheme of the measuring system was consistent with the scheme presented in fig. 4. The illumination intensity in the image plane was recorded with the help of a photoresistor, a multistage amplifier and X-Y plotter.

1. The illumination intensity at a given point of the shining background produced by the bubble is proportional to the surface ΔS of the shining surface element.

The illumination intensity has been measured twice for a chosen point in the shining background produced by the bubble and the following results have been obtained (fig. 7):

– For a surface ΔS_1 equal to π cm² an amount of 170 conventional units have been obtained.

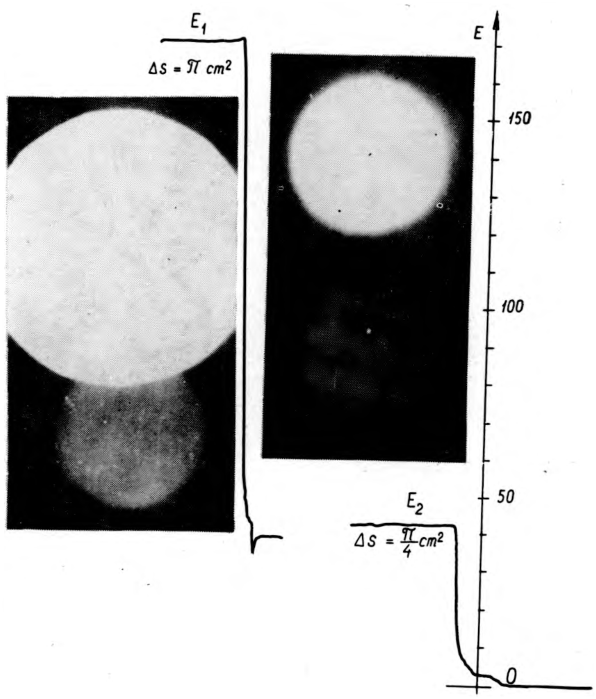

Fig. 7. The dependence of the illuminating intensity E in the randomly choosen point upon the area of the object surface ΔS

– For a surface ΔS_2 equal to $\pi/4$ cm² an amount of 42.5 conventional units have been obtained

$$\frac{E_1}{E_2} = \frac{\Delta S_1}{\Delta S_2} = 4.$$

From the photographs associated with the graph (fig. 7) it follows additionally that the shining background area does not depend on the ΔS element area. It should be noted that

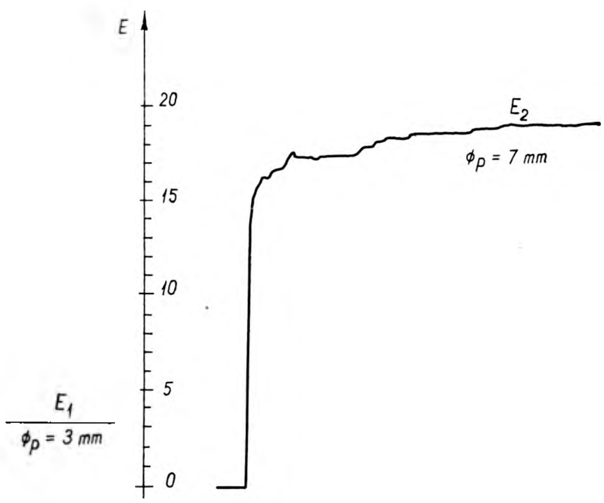

Fig. 8. Dependence of background illumination intensity upon the bubble diameter

it does not depend on the bubble diameter either.

2. The intensity of illuminations within the shining background area of a bubble is directly

Fig. 9. An example of the background region dependence upon the bubble position in the optical system

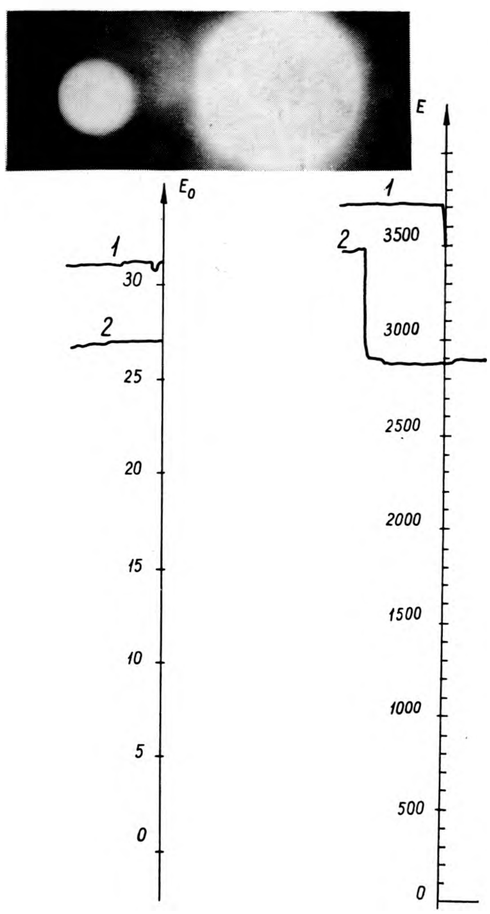

Fig. 10. An example measurement of the relative intensity measurement E_w in the background due to bubble. The value estimated from (8) is $E_w = 1/120$

1. $E_w = \frac{1}{117}$; 2. $E_w = \frac{1}{128}$

poportional to the surface of the bubble cross-section. In a point randomly chosen within the shining backgroun l the intensity of illumination has been measured twice for the bubbles of diameter 7 mm and 3 mm. In view of (8) it may be expected that the illumination intensity is changed by the factor 5.44.

From the graph presented in fig. 8 it may be concluded that

$$\frac{E_2}{E_1} = 5.76,$$

which approximately satisfies the requirements of the formula (8).

3. The magnitude of the image background due to a bubble depends on the distance l of the bubble from the objective (fig. 9, case of two bubbles).

4. Relative intensity in the background area. The results of two measurements of the maximal illumination intensity in the image plane is presented in fig. 10. The value evaluated from (8) amounts to $E_{w\max} = 1/120$, which is consistent to a sufficient degree with the measured values.

Влияние пузырей в элементах оптических инструментов на качество отображения

В статье предпринята первая попытка приближённой оценки распределения освещённости плоскости изображения в оптических инструментах, вызванной рассеянным светом на пузырях. Полагают, что результаты работы могут быть рациональной основой для допуска пузырчатости оптических элементов.

References

[1] Davis G. E., *Scattering of Light by an Air Bubble in Water*, JOSA 45, 7, pp. 572–581 (1955).

[2] Born M., Wolf E., *Principles of Optics*, Pergamon Press, New York 1968.

Received, November 23, 1976