One approximation to multiple beam amplification in negative Kerr-type media

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It is shown that the interaction of two strong pumps with four weak signal beams, not coplanar with the pump beam, gives rise to the exponential amplification of the weak beams in a Kerr-type negative medium. The theory developed is applied to the case of noise formation in iron-doped lithium niobate.

Keywords: nonlinear optics, photorefractive medium, isotropic Kerr medium.

1. Introduction

Some wave mixing processes in photorefractive and other nonlinear media lead to exponential amplification of weak signal beams as a result of beam interaction with a strong pump beam. These processes can limit the crystal performance since noise is generated. The photoinduced noise is a nonlinear process and is the result of beam-coupling when a seed radiation is amplified at the expense of the incident beam. Up till now, the exact causes of the noise have not been firmly established.

For two wave mixing, an exponential gain is possible if the medium response is not local (diffusion mechanism) or for nonstationary conditions, *i.e.*, in which a $\pi/2$ phase shift takes place between an interference pattern and gratings. When a laser beam is incident normally upon the crystal and perpendicularly upon the crystalline *c*-axis, a strong steady state symmetrical noise appears. The steady-state three wave mixing process can produce gain in positive (self-focusing) Kerr-type media with local response. In photorefractive media, this is realized when a strong external electric field is applied, for example, in the photorefractive strontium barium niobate (SBN) crystal [1]. In iron-doped lithium niobate (LiNbO$_3$) crystals, it is well known that nonlinearity is local and negative, thus the three-wave mixing process does not

explain the characteristic noise observed in this case. The argument of KAMBER et al. [2] and GUOQUAN ZHANG et al. [3], which contain interesting experimental data, is based on the hidden assumption of positive nonlinearity. Because of this [4], the noise in LiNbO$_3$ is produced, at least partly, by nonstationary process. Contributions of four--wave mixing processes were discussed in [5]. The important part of amplification produced can be explained by four-wave mixing only by additionally assuming a frequency detuning of signal waves. Here, we demonstrate that the stationary mixing of six waves (with at least two strong ones) can, additionally to the already known mechanisms, produce exponential gain in negative Kerr-type media. We discuss possible contribution of this process to the noise in iron-doped lithium niobate. First, we discuss the isotropic Kerr medium as the simplest model and after this the changes which are introduced by an anisotropy characteristic for lithium niobate.

2. Isotropic case

We assume the general geometry, as depicted in Fig. 1. There are two strong orders S_0, S_1, and four weak ones S_2 to S_5, the wave function is taken as a sum of six diffraction orders, according to the following equation:

$$\psi(x, y, z) = \sum_{p=0}^{5} S_p(z) \exp\left(iK_{p,x}x + iK_{p,y}y\right) \tag{1}$$

In the paraxial approximation, the wave amplitude evolution along the propagation coordinate z is taken as:

$$i2k\frac{\partial \psi}{\partial z} = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} - 2k^2\frac{\Delta n}{n_0}\psi \tag{2}$$

where x, y are the transversal coordinates, k is the wave vector in a medium, n_0 is the average refractive index of a medium, and $\Delta n(x, y, z)$ is the light-induced refractive index change (see Fig. 1).

First, an isotropic self-focusing Kerr medium is considered, for which $\Delta n = -n^2|\psi|^2$, where $n^2 (>0)$ is the Kerr coefficient. In a normalized form, the propagation equation is:

$$i\frac{\partial \psi}{\partial z'} = -\frac{1}{2}\left(\frac{\partial^2 \psi}{\partial x'^2} - \frac{\partial^2 \psi}{\partial y'^2}\right) + \kappa|\psi|^2\psi \tag{3}$$

with a positive nonlinear coefficient $\kappa = n_2/n_0$ and dimensionless coordinates $x' = xk$, $y' = yk$, and $z' = zk$. We assumed the simplest linear approximation, for which it is supposed that the strong beams are undeleted and their propagation is the same as in
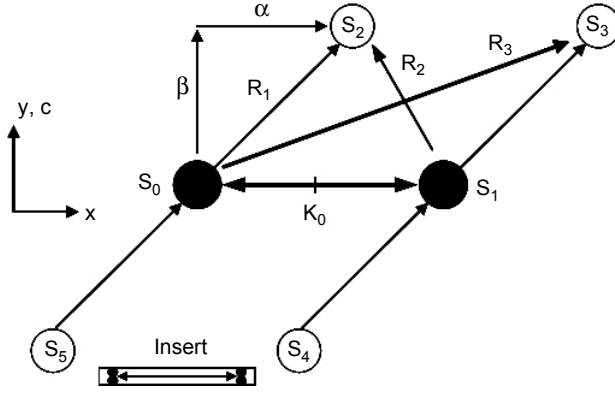
Fig. 1. Wave vector projections on the transversal plane for the interaction geometry. Two strong pumps S_0 and S_1 are separated by wave vector $2K_0$. The orders S_2–S_5 are weak signal beams. For the anisotropic case every pump beam is considered as two beams of half intensity with small separation along y-axis which we take parallel to the crystal c-axis (insert).

the absence of weak beams. If Bragg regime approximation is valid ($\kappa|\psi|^2 \ll K_0^2$), it can be written:

$$\psi_0 = S_0(z')\exp(-iK_0x') + S_1(z')\exp(iK_1x') \tag{4}$$

and, after substituting in Eq. (3) and neglecting higher diffraction orders, for order evolution:

$$S_{0,1}(z') = S_{0,1}(0)\exp\left[-\frac{iK_0^2}{2} - ik\left(I_{0,1} + 2I_{1,0}\right)\right]z' \tag{5}$$

with beam intensities $I_{0,1} = |S_{0,1}|^2$. In the following, we assume that $S_0(0) = = S_1(1) = 1$, and neglect terms proportional to the squares of weak orders. We approximate Fourier harmonics for light intensity distribution (shown in Fig. 1) as:

$$|\psi|^2 = R_K(z')\exp\left[i\left(K_xx' + K_yy'\right)\right] \tag{6}$$

The directions of wave vectors are determined by parameters α and β (Fig. 1).

The linear harmonics in weak order amplitudes are (we omit the z-dependence):

$$R_1 = S_2S_0^* + S_3S_1^* + S_1S_4^* + S_0S_5^* \tag{7}$$

$$R_2 = S_2S_1^* + S_0S_4^* \tag{8}$$

$$R_3 = S_2S_0^* + S_1S_5^* \tag{9}$$

The two strong harmonics are:

$$R_0 = S_1 S_1^* + S_0 S_0^* \tag{10}$$

$$R_{2\kappa} = S_1 S_0^* \tag{11}$$

We make the substitution:

$$S_2(z') = S_2^*(z') \exp\left[-i\left(\frac{K_0^2}{2} + 3\kappa\right)z'\right] \tag{12}$$

and similar equations for amplitudes S_3–S_5 to eliminate explicit exponents. A system of linear equations is obtained if we use Eq. (1) as wave function and Eqs. (6)–(12) for the intensity distribution in Eq. (3) by neglecting the squares of weak order amplitudes:

$$i\partial_{z'} S_2' = g_1 S_2' + \kappa\left(S_2' + 2S_3' + 2S_4'^* + S_5'^*\right) \tag{13}$$

$$i\partial_{z'} S_3' = g_2 S_3' + \kappa\left(S_3' + 2S_2' + 2S_5'^* + S_4'^*\right) \tag{14}$$

$$i\partial_{z'} S_4' = g_1 S_4' + \kappa\left(S_4' + 2S_5' + 2S_2'^* + S_3'^*\right) \tag{15}$$

$$i\partial_{z'} S_5' = g_2 S_5' + \kappa\left(S_5' + 2S_4' + 2S_3'^* + S_2'^*\right) \tag{16}$$

where $g_{1,2} = (1/2)(\alpha^2 + \beta^2 \mp 2K_0\alpha)$.

Two particular cases with exponential growing solutions can be easily identified. The first case is when $g_2 = 0$ which corresponds to the wave vector of the weak beam on the circle (Fig. 2a). If we assume that S_5 and S_3 remain weak upon propagation, the combination $S_2 - S_4$ has a growing solution proportional to $\exp(\sqrt{3}\,\kappa z')$. This case
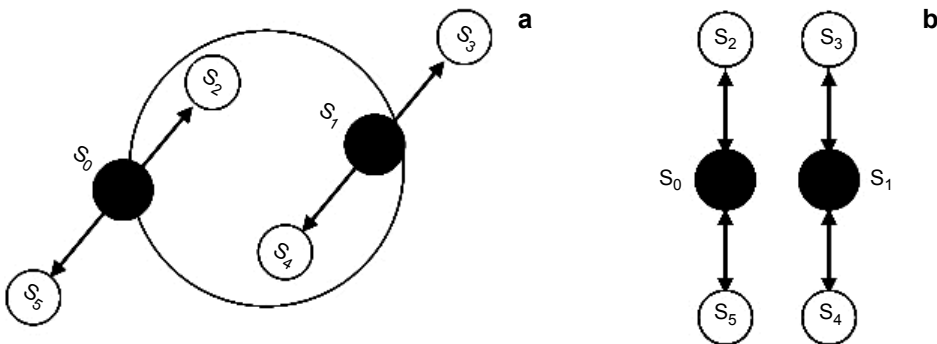

Fig. 2. The two wave vector configurations producing gain, the "ring" (**a**) and the "line" (**b**).

was discussed in [5]. The second possibility is having $\alpha = 0$, which corresponds to two straight lines (Fig. 2**b**). The growing combination in this case is $S_2 + S_3 - S_4 - S_5$ and the growth is proportional to $\exp(z' \sqrt{\beta^2 \kappa - \beta^4/4})$. It is seen that the second case is quite similar to the modulation instability type amplification. The gain exists only if $\beta^2 < 4\kappa$. The maximal gain equal to κ is obtained for $\beta^2 = 4\kappa$. Thus, the "ring" amplification of geometry of Fig. 2**a** is $\sqrt{3}$ times stronger than the maximum "line" amplification of Fig. 2**b**.

To investigate the gain distribution on the plane of possible wave vectors, we have performed a numerical solution for propagation. The results are presented in Figs. 3 and 4. It is seen that the two above-mentioned particular cases give a good approximation for two regions where the gain is obtained (see also Fig. 5). The quarter of the circle with negative α numbers in Fig. 3**a** has a weaker amplification than
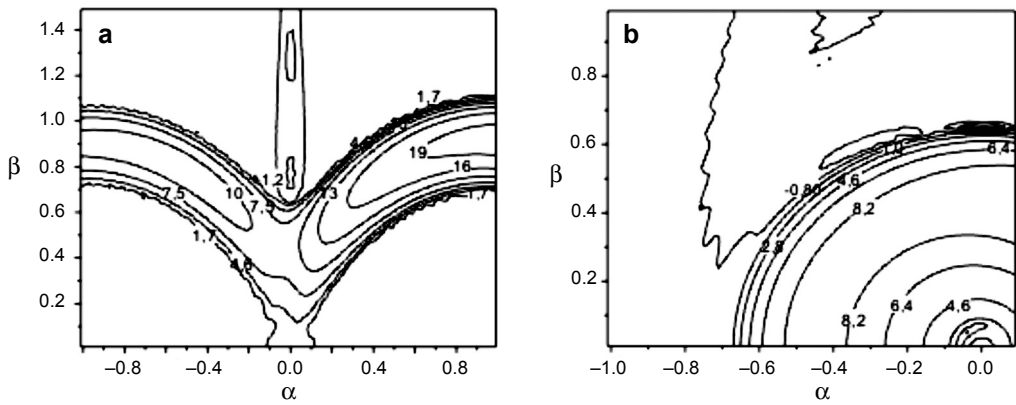

Fig. 3. The numerical calculation of weak beam amplification in a plane of parameters α–β for isotropic case. Initially, a single weak beam S_2 is taken and the propagation over distance $L = 60$ mm is calculated for $\kappa = 0.1$. The lines correspond to regions of equal gain $g = \ln[I_2(L)/I_2(0)]$; $K_0 = 1$ (**a**), and $K_0 = 0.1$ (**b**).
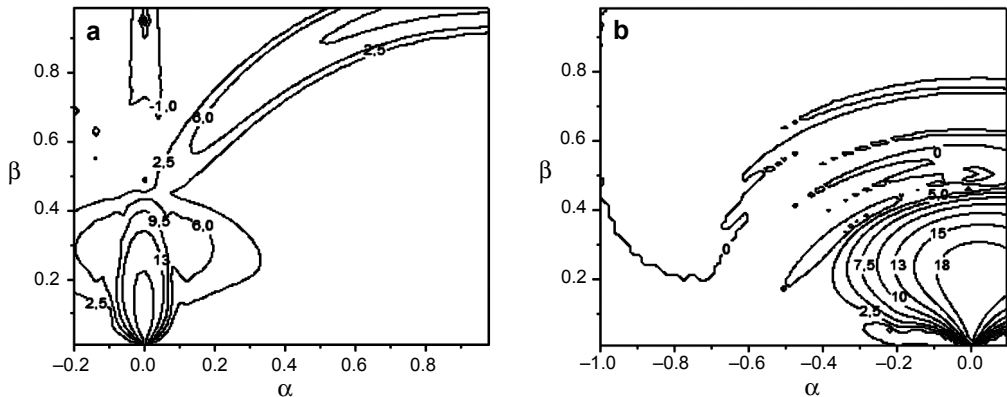

Fig. 4. The same as in Fig. 3 for anisotropic case, $L = 60$ mm, $\kappa = 0.1$; $K_0 = 1$ (**a**), and $K_0 = 0.1$ (**b**).

the quarter with positive α. The negative α part corresponds to the amplification of a pair S_2 and S_5. For a bigger K_0/κ ratio, the difference between gains in a pair becomes bigger, too, and the amplification pattern concentrates on the circle passing through two pumps.

The gain is produced as a result of modification of propagation phase velocity due to nonlinear interaction. The magnitude of this modification is of κI order. Thus, the characteristic width of amplification regions in the κ-plane is determined by the condition $\Delta k_z \leq \kappa I$, where Δk_z is the difference of linear propagation wave vector in z-direction for strong and weak beams.

If the angle between pumps becomes smaller, the gain concentrates on a ring, with a characteristic size corresponding to the "line" amplification mechanism (Fig. 3**a**), but the approximation of two pump beams is not good for small angles in the isotropic material, because higher diffraction orders of the pump beams are produced.

3. Anisotropic case

Now, we consider the case of a self-defocusing optical medium: $\chi^{(3)} < 0$, $n_2 < 0$. The nonlinearity in photorefractives generally demonstrates strong saturation, and formally can be approximated by a Kerr nonlinearity only for a limit of a small effective contrast of interference pattern (*e.g.*, for additional incoherent illumination or big dark conductivity). Nevertheless, if higher harmonics of the space charge field are neglected, the purely Kerr nonlinearity gives a reasonable model for interaction of plane waves with recalculation of coefficients. In photorefractive media there is also a strong anisotropy of nonlinearity. Thus, to the simplest approximation, the spatial harmonics for the refractive index change can be written as:

$$\Delta n = -n_2 \sum_{\kappa} \frac{R_{\kappa}(z) K_x^2}{K_x^2 + K_y^2} \exp\left(i K_x x + i K_y y\right) \tag{17}$$

This equation takes into account that interference fringes which are not perpendicular to the crystal c-axis and write less efficient gratings. Proceeding in the same way as in the isotropic case it is possible to see that the two pump beam configuration gives a gain in the ring (Fig. 2**a**), but not on a line (Fig. 2**b**). This contradicts the experimental observations. In fact, to obtain a more realistic picture, it is necessary to take into account the structure of two pump beams, as indicated in Fig. 1.

The reason for this is the following: for the isotropic case the nonlinear interaction between two pumps modifies their phase velocities. When there is a weak signal beam, the modification of phase velocity for this beam is not the same as for the strong pumps. This gives a synchronization which is necessary for gain. For the geometry of Fig. 1 in the lithium niobate case, the two pumps do not write a grating. Nevertheless, the nonlinear modification of phase velocities still takes place because of the internal structure of each pump. One can imagine that the beam writes its own waveguide and this modifies its phase velocity. To take this into account, we suggest that each of

the pumps, in fact, consists of two nearly collinear plane waves, as shown in the insert of Fig. 1. The computer calculation of gain distribution for this case in the undeleted pump approximation is shown in Fig. 4. When two pump beams are well separated in the *k*-plane, the amplification concentrates on a vertical line, and on a circle. The maximum gain for the line is approximately two times bigger than for the circle (Fig. 4**a**). When the pump beams become closer, the gain distribution becomes wider (Fig. 4**b**) and has a characteristic shape of two spots elongated along the *c*-axis. Note that the values of gain, as given in Fig. 4, are relative and the estimation of validity of absolute numbers proceeds just by noting that the gain coefficient has an order of nonlinear coefficient κ. Thus, the value is similar to gain values obtained with other models, and the discussion of [2, 3, 5] equally applies.

4. Experimental data

The characteristic pattern of noise produced by two beams in $LiNbO_3$, which consists of a ring and two lines, is well known from the preliminary experiments in this material [6].

To investigate the transition from the two lines and a circle to a single beam noise spot, experiments have been performed with the configuration shown in Fig. 5. The two beams derived from a solid-state green laser (50 mW, 532 nm) were crossing inside the 1 mm thick and 3×3 mm^2 front face iron-doped lithium niobate crystal. The iron content of our crystal is 0.03 wt%. Characteristic patterns of the output noise are shown in Fig. 6. We have measured the angular noise distributions for different angles between the pump beams. It is seen that when the angle between pump beams becomes smaller the noise lines become wider, but the characteristic angle corresponding to maximal gain in the vertical direction does not change much. The ring in our experiments was seen only for the initial stage of recording and visibly
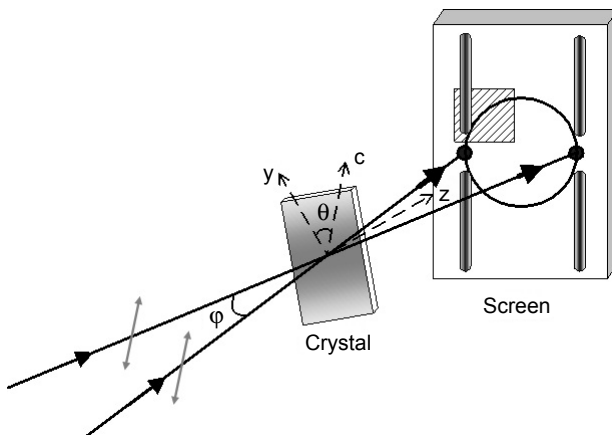

Fig. 5. The observation geometry. The dashed region on the screen corresponds to the calculation region in Figs. 3 and 4. The propagation axis *z* is parallel to the sum of *k*-vectors for two pump waves.

disappears later. We explain this by a relatively low doping level and thin sample. The results of [5] for thicker and more heavily doped sample also demonstrate strong initial amplification for the ring, and bigger steady-state "line" amplification than the "ring" one.
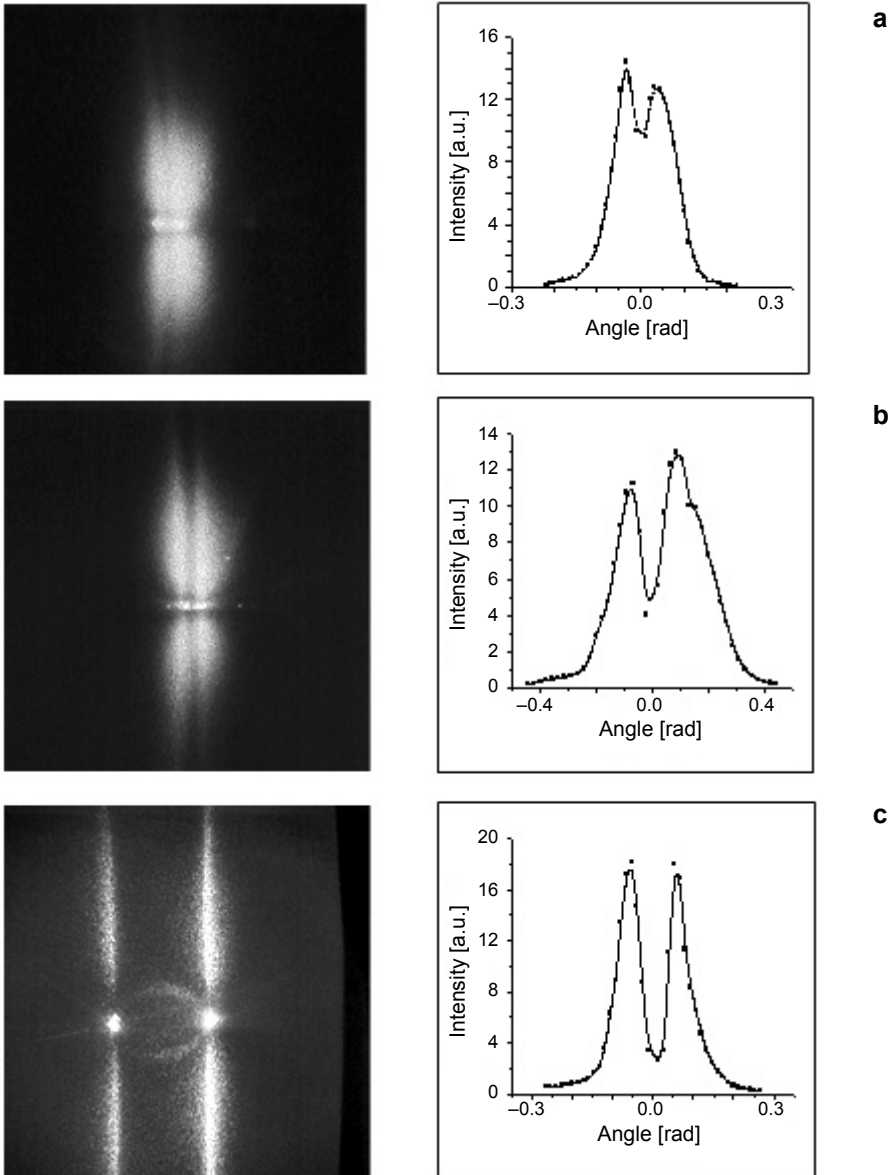

Fig. 6. Photographs and profiles of the angular noise distribution using lithium niobate crystal for different angles between two pump beams (14 mrad, 70 mrad and 34 mrad). The profiles are measured along vertical line corresponding to the c-axis direction and passing across the pump beam direction. The pump beam intensity (for zero angle) is not shown (see Fig. 7).
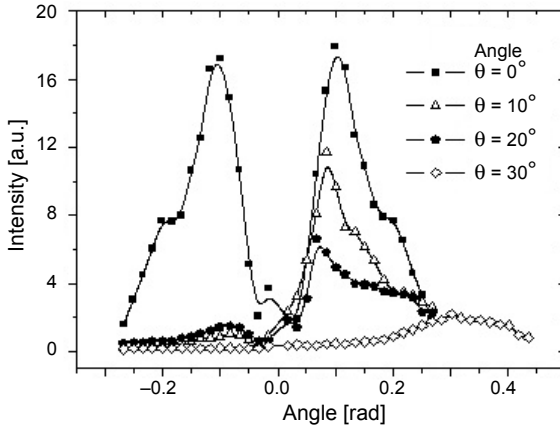
Fig. 7. Profiles of the angular noise distribution using lithium niobate crystal for the crystal *c*-axis not perpendicular to the main propagation direction *z*. The angle between *c*-axis and transversal direction *y* is equal to *θ*, *φ* = 34 mrad (Fig. 5).

If the crystal is rotated in such way that the *c*-axis moves in *y–z* plane (Fig. 7), the distribution becomes asymmetric and, for the angle between crystal *c*-axis and *y*-direction being big enough, the noise strongly diminishes and the most important part of it is displaced to bigger angles. This is related to the crystal birefringence. In our previous work [4], we have suggested that the momentum conservation still holds for this case, but more careful calculation demonstrates that if birefringence exists the noise intensity distribution can become asymmetric.

5. Discussion and conclusions

It is shown that multiple waves mixing in negative Kerr-type media can produce exponential gain, but the necessary configuration is quite complicated and includes at least two pump beams and weak beams which are not coplanar with the pumps. The two-pump configuration cannot produce gain; for two-pump configurations, with coplanar pump and signal beams, the direct proof is unknown in this work; however, numerical experiment suggests that the exponential gain is impossible yet [7].

The four-wave mixing with amplification on a straight line is possible only for the nonstationary mechanism [5].

Thus, the six-beam configuration discussed here seems to be the simplest one, where the exponential gain with angular distribution similar to the one experimentally observed for the modulation instability type process exists (in negative Kerr-type medium with local response and in a steady-state condition). The four-wave mixing configuration of [5] does not demonstrate gain for steady state (though such gain is possible for small frequency detuning). On the other hand, the Bragg conditions for modulation instability type process do not work, and correct description generally must include conjugate waves. For "ring" amplification, conjugate waves can be weak, but

for the "line" type, which is of the main interest for noise formation, the conjugate waves have the magnitude similar to other signal waves.

The instability of a single fringe (1D dark soliton embedded in 2D) for negative nonlinearity was predicted theoretically [8] and observed in photorefractive crystals and rubidium vapor [9, 10]. The geometry we consider here can be looked at as an array of dark fringes. The growing combination of noise beams, which correspond to the "line amplification", in fact, gives rise to a "snake" instability, characteristic of a dark soliton breaking.

The angular dependence of gain given by the formula $\exp(z'\sqrt{\beta^2\kappa - \beta^4/4}\,)$ is quite similar to the one for more usual modulation instability mechanism. The main difference between the two equations is the somewhat smaller angle corresponding to the maximum gain for negative nonlinearity and smaller value of the gain itself. The exact nonlinearity values for lithium niobate are difficult to obtain, and higher diffraction orders can change the angular intensity distribution.

Our model suggests that the basic noise formation mechanism for small-angle scattering in lithium niobate is related to a two-pump configuration, and not to a single pump. The noise amplification produced in the presence a pump beam develops scenarios of a finite spatial spectrum for this case. Thus, the realistic calculation in this case must model the propagation of a beam with essential 2D transversal structure, which is a numerically intense task. The big angle noise seen when the crystal is tilted (Fig. 7) is not explained by the theory presented and is most probably due to the nonstationary amplification mechanisms suggested earlier [4]. It is also possible that nonstationary effects can enhance gain for six-wave mixing, as well.

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