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Study on power efficiency of vortex beam propagation through an optical system with phase optimization

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Beam blocked and truncated by the receiver causes serious power losses in beam uplink propagation in a relay mirror system. We propose a method to improve power efficiency of beam uplink propagation in the relay mirror system by using vortex source and phase optimization. A typical model of beam uplink propagation in the relay mirror system is established. With this model, the principle of the method is theoretically analyzed, and power efficiencies of beam uplink propagation under different conditions are calculated. The calculation results show that power efficiency of beam uplink propagation can be improved from 86.44% to 97.86% by using vortex source and phase optimization. A reduced-scale experiment of beam uplink propagation in the relay mirror system under the "closed-loop" mode is performed, and the experimental results show that power efficiency can be improved from 71.89% to 91.59% by using the vortex source and phase optimization.

Keywords: power efficiency, relay mirror system, vortex beam, phase optimization.

1. Introduction

In recent years, the relay mirror technology has become a lively area of scientific research [1–8]. The relay mirror technology is an important system combat concept which was first gained currency in the 1980's, during Ronald Reagan's "Star Wars" anti-missile program [1]. A relay mirror system is composed of the laser source, the launching module, the relay platform and other relevant units [2]. The relay platform can be located on an aircraft, unmanned air vehicle (UAV), high-altitude airship (HAA) or space-based platform. The source can be the high-energy lasers on a ground, sea or aircraft platform. The source beam is launched to the relay platform, and the uplink beam is received by a Cassegrain telescope at the platform. The received-uplink beam is then redirected to the target after being cleaned up [3, 4]. The relay platform provides a cooperative beacon in the "source-to-relay" uplink, which can be used to sense atmospheric aberrations in real time and minimize them with adaptive optics. The relay mirror system separates the laser source from the beam director, allowing each subsystem to operate in its most advantageous environment. Thus, the relay mirror system is considered to have advantages, *e.g.*, it can reduce the influence of atmosphere, extend range to target, lower system tracking bandwidth, make integration time longer and lower laser illuminator power [5].

The Air Force Research Laboratory and the Boeing Company have performed a set of important experiments since the 1990's. In 1990, the Air Force Research Lab performed the Relay Mirror Experiment (RME) on the island of Maui, which successfully targeted a ground based laser on an orbiting satellite, then reflected the laser radiation to another ground facility [6]. RME demonstrated that a laser beam could be propagated from a ground based site through the Earth's atmosphere, to a satellite, track the satellite and be "bounced" back to a target board [6]. RME was the first step in proving the feasibility of a relay mirror and its utility in the military framework. In the late 1990's, a study performed by the Air Force Research Laboratory recognized potential missions for a space based optical relay mirror. In 2000, a preliminary satellite design was completed by a team of Naval Postgraduate School master's students resulting in a scissor-like Bifocal Relay Mirror spacecraft. The spacecraft consists of two optically coupled telescopes including adaptive optics used to redirect the light from a ground based laser to a distant target [7]. In 2006, Boeing demonstrated Aerospace Relay Mirror System (ARMS) at Kirtland Air Force Base, New Mexico [8]. The ARMS which was a half-scale version of a strategic relay mirror, successfully redirected a laser beam to a target two miles away. The experiment demonstrated that a relay mirror system can receive laser energy and redirect it to a target, extending the laser's range. But the experimental results showed that power efficiency of the ARMS was merely 50%, beam blocked and truncated by the receiving Cassegrain telescope caused serious power losses [9, 10]. In recent years, it has been widely investigated how to improve power efficiency of the relay mirror system. MANSELL has analyzed beam shaping for relay mirrors [11], PU ZHOU *et al.* have analyzed the possibility of improving power efficiency of the relay mirror system by using coherent laser array [12]. In the present paper, we report a method to improve power efficiency of beam uplink propagation in the relay mirror system by using vortex source and phase optimization. We analyze the principle and perform a reduced-scale experiment.

2. Models and theoretical analysis

2.1. Relay mirror system

A typical model of the relay mirror system is shown in Fig. 1, and schematic diagram of beam uplink propagation in a relay mirror system is shown in Fig. 2. A Cassegrain telescope with central obscuration is used as the uplink beam receiver.

The relay mirror system employs a cooperation beacon and two adaptive optics (AO) systems. As shown in Fig. 1, AO1 that is located at the ground is used for phase pre-correction in uplink propagation, and AO2 that is located at the platform is used

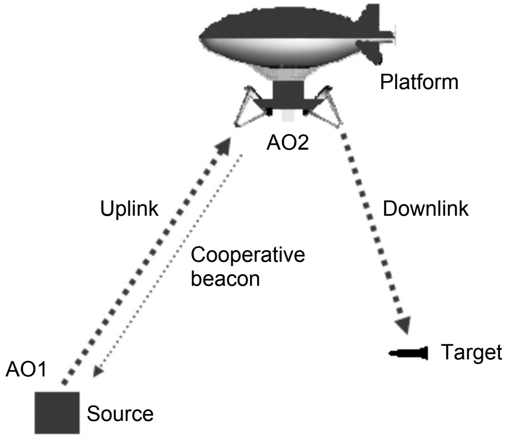

Fig. 1. Model of a relay mirror system.

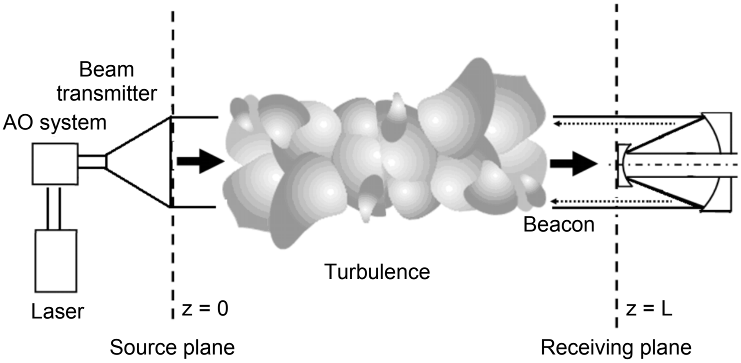

Fig. 2. Schematic diagram of beam uplink propagation in a relay mirror system.

for phase cleaning of the received uplink beam. By using the cooperative beacon, the system can sense the atmospheric distortions in the "source-to-relay" uplink in real time. As described in reference [2], if the AO system does a good job of correcting the phase, the resulting performance will be near the diffraction limit. Phase correction of AO1 $\psi(\varepsilon, \eta)$ can be expressed as

$$\psi(\varepsilon, \eta) = \begin{cases} -\phi(\varepsilon, \eta) & \text{closed-loop} \\ 0 & \text{open-loop} \end{cases} \tag{1}$$

where $\phi(\varepsilon, \eta)$ denotes the turbulence phase-screen at the source plane, which can be detected in real time by using the cooperative beacon. By using the Huygens–Fresnel principle, optical field at the receiving plane $U(x, y, z)$ can be expressed as

$$U(x, y, z) = \frac{e^{jkz}}{j\lambda z} \iint U_0(\varepsilon, \eta) e^{j\phi(\varepsilon, \eta)} e^{j\psi(\varepsilon, \eta)} e^{j\frac{k}{2z}\left[(x-\varepsilon)^2 + (y-\eta)^2\right]} \mathrm{d}\varepsilon \mathrm{d}\eta \tag{2}$$

where $U_0(\varepsilon, \eta)$ denotes the optical field at the source plane. Power efficiency of the beam uplink propagation is defined as

$$\eta = \frac{\int_S I(x, y, z)\mathrm{d}s'}{\int_0^\infty I(x, y, z)\mathrm{d}s'} \tag{3}$$

where $I(x, y, z)$ denotes optical intensity distribution at the receiving plane, S denotes the window area of the receiving telescope.

2.2. Vortex beam

Vortex beam is a laser beam with continuous spiral phase distribution, which can be generated by using holographic gratings, vortex phase plates, beam mode converters and resonators with phase converters [13–16]. Phase distribution of a vortex beam is shown in Fig. 3.

Optical field of the vortex beam can be expressed as

$$U_0(r, \theta) = U_0(r)\exp\left[i\varphi(\theta)\right] \tag{4}$$

$$\varphi(\theta) = n\theta \tag{5}$$

where (r, θ) denotes the coordinates, $U_0(r)$ is the amplitude distribution, $\varphi(\theta)$ is the phase distribution, and n is the optical topological charge of the beam. The characteristics of the vortex beam have been widely investigated in recent years. Vortex beam has wide applications in many areas due to its unique characteristics, including optical micromanipulation, atomic optics, biomedicine nonlinear optics, and the optical transmission of information [13–17]. One of the most important unique characteristics is a dark core at the center of the beam carrying optical vortex. The size

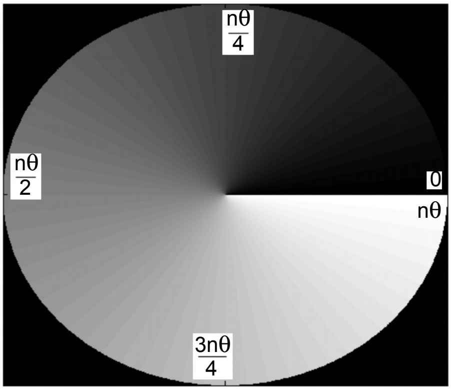

Fig. 3. Phase distribution of a vortex beam.

and shape of this dark core can be adjusted by changing the topological charge of the optical vortex [18]. Thus, in a relay mirror system with a vortex source, power losses in uplink propagation can be decreased by optimizing the optical topological charge. The dark core at the receiving plane is adjusted to be matched with the secondary mirror of the receiving Cassegrain telescope to the greatest extent.

2.3. Phase optimization

In beam uplink propagation process in the relay mirror system, optical intensity distribution at the receiving plane can be shaped by controlling phase distribution of the source, and power losses in uplink propagation can be decreased by optimizing phase distribution at the source plane [19]. The phase optimization process can be combined with adaptive optics corrections in the relay mirror system [11, 19]. In the present paper, we use the stochastic parallel gradient descent algorithm to optimize phase distribution of the source. The control loop can be described as the following steps [20]:

1) Define power efficiency of the uplink beam η as the evaluation function of the optimization process.

2) Define samples number of phase distribution at the source plane M and cycle index of the optimization process m. Set $\psi_i = 0$, $i = 1, 2, ..., M$.

3) Generate statistically independent random perturbations $\delta\psi_i$, $i = 1, 2, ..., M$, all $|\delta\psi_i|$ are small values that are typically chosen as statistically independent variables having zero mean and equal variances, $\langle\delta\psi_k\rangle = 0, \langle\delta\psi_k\delta\psi_l\rangle = \sigma^2\delta_{kl}$, where δ_{kl} is the Kronecker symbol.

4) Compute the difference between two evaluations of the relay mirror system performance $\delta\eta = \eta^+ - \eta^-$, where $\eta^+ = \eta(\psi_1 + \delta\psi_1, \psi_2 + \delta\psi_2, ..., \psi_M + \delta\psi_M)$ and $\eta^- = \eta(\psi_1 - \delta\psi_1, \psi_2 - \delta\psi_2, ..., \psi_M - \delta\psi_M)$.

5) Update the value of ψ_i according to the equation $\psi_i^{t+1} = \psi_i^t + \gamma\delta\psi_i^t\delta\eta$, where t denotes the steps of iteration, γ is the update gain, $\gamma > 0$ accords to the procedure of maximization.

6) Repeat steps 3–5 until algorithms converge and the optimal optical phase distribution of the source can be obtained.

2.4. Methods

We propose for improving a method power efficiency of beam uplink propagation in the relay mirror system by using vortex source and phase optimization. The main motivation behind this method is to decrease power losses which were induced by blocking and truncation of the receiver. The method includes two parts: in the first step, a vortex beam is used as the source. Optical topological charge of the vortex source is optimized to let the dark core at the receiving plane be matched with the secondary mirror of the receiver to the greatest extent. Secondly, phase distribution of the vortex source is optimized to reduce power losses further, which is controlled by using a stochastic parallel gradient descent algorithm.

3. Numerical calculations

We analyze power losses of the uplink beam in a relay mirror system under different conditions. Parameters of the system are set as: truncation aperture of the source beam is 1.2 m, wavelength of the source is 1.064 μm, uplink propagation distance is 30 km, the zenith angle of the propagation path is π/4, the uplink receiver is a Cassegrain telescope with 1.2 m-aperture main mirror and 0.3 m-aperture secondary mirror, distribution of the structure constant of the atmosphere is described as the H-V 5/7 turbulent model [19]

$$C_n^2(h) = 8.2\times 10^{-56} V(h)^2 h^{10}\exp\left(\frac{-h}{1000}\right) + 2.7\times 10^{-16}\exp\left(\frac{-h}{1500}\right) + C_0\exp\left(\frac{-h}{100}\right) \tag{6}$$

$$V(h) = 5 + 30\exp\left[-\left(\frac{h-9400}{4800}\right)^2\right] \tag{7}$$

where h is the altitude from the ground, $V(h)$ is the wind speed along the vertical path, C_0 is the nominal value of at ground level (the typical value is 4.0×10^{-14} $\mathrm{m}^{-2/3}$). In the present paper, we use the Zernike polynomials to calculate the turbulence phase-screen in beam uplink propagation [21].

3.1. Relay mirror system with flat beam source

At first, we analyze power losses of beam uplink propagation in a relay mirror system with a flat beam source. The source is a flat beam with unit amplitude and zero initial phase distribution. After calculation, we get optical intensity distribution at the receiving plane under the “open-loop” mode, as shown in Fig. 4**a**, and optical intensity distribution at the receiving plane under the “closed-loop” mode, as shown in Fig. 4**b**, and optical power distribution at the receiving plane under different AO modes given in Tab. 1.

Table 1. Power distribution at the receiving plane.

AO modes	Power blocked by the secondary mirror	Power out of the main aperture	Power in the receiving area
Open-loop	6.56%	44.27%	49.17%
Closed-loop	7.74%	5.82%	86.44%

We see that power efficiency of the beam uplink propagation in the relay mirror system with a flat beam source is low. Under the “open-loop” mode, power efficiency is merely 49.17%. Under the “closed-loop” mode with turbulence completely corrected, power efficiency is 86.44%. In uplink propagation, the beam spot spreads, with

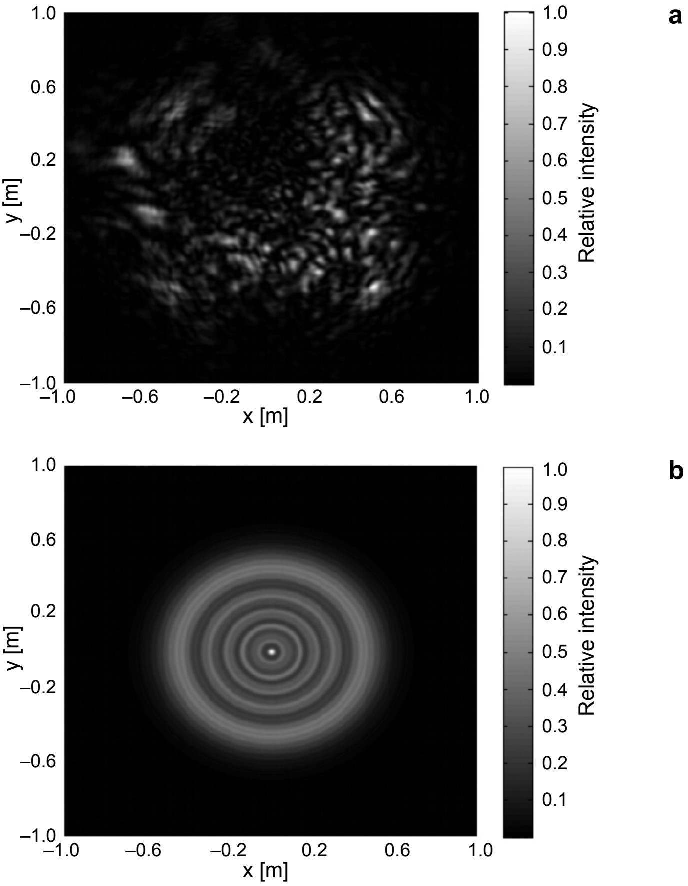

Fig. 4. Intensity distribution at the receiving plane under the "open-loop" mode (**a**) and under the "closed-loop" mode (**b**).

intensity gathered at the center. Thus, the beam blocked and truncated by the receiving telescope results in serious power losses.

3.2. Relay mirror system with vortex source

We analyze power losses of the uplink beam in a relay mirror system with a vortex beam source. Optical topological charge of the vortex source is optimized to ensure that the dark core at the center can be matched with the secondary mirror of the receiver to maximum. Firstly, we calculate power efficiencies of beam uplink propagation in vacuum condition (AO "closed-loop" mode) with different optical topological charges, the results of which are shown in Fig. 5.

The results show that the optimal value of topological charge is $n = 5$. We take $n = 5$ and then calculate power efficiency of beam uplink propagation in the relay

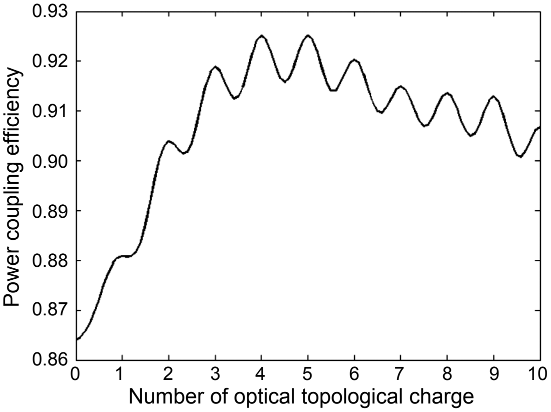

Fig. 5. Power efficiencies of beam uplink propagation with different values of source optical topological charges.

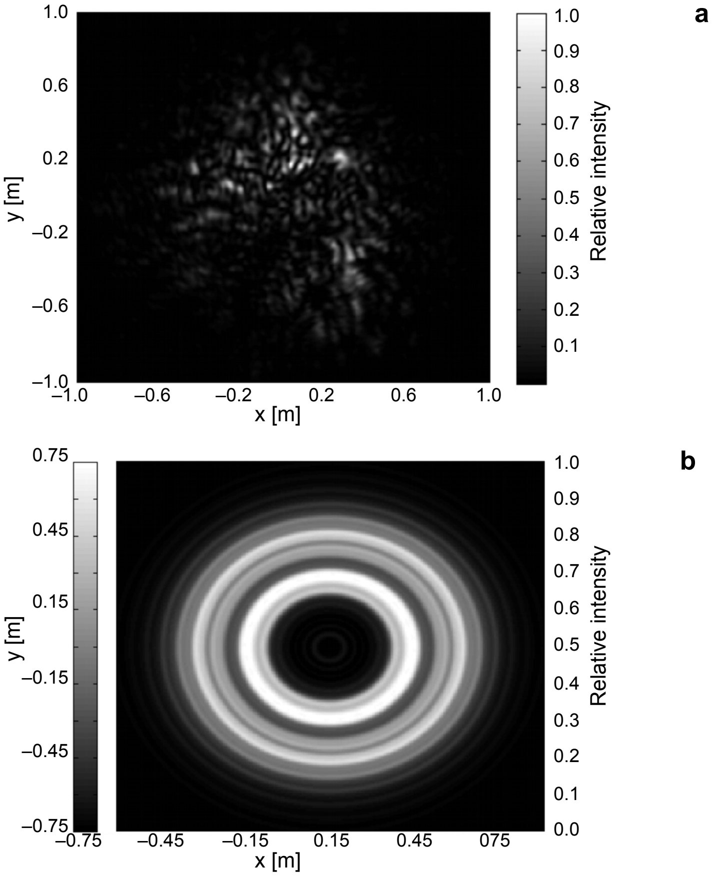

Fig. 6. Intensity distribution at the receiving plane with vortex source under the “open-loop” mode (**a**) and under the “closed-loop” mode (**b**).

Table 2. Power distribution at the receiving plane with vortex source.

AO modes	Power blocked by the secondary mirror	Power out of the main aperture	Power in the receiving area
Open-loop	5.21%	27.34%	67.45%
Closed-loop	0.83%	6.64%	92.53%

mirror system with vortex source. Optical intensity distribution at the receiving plane under the "open-loop" mode is shown in Fig. 6**a**, and optical intensity distribution at the receiving plane under the "closed-loop" mode is shown in Fig. 6**b**. Table 2 shows optical power distribution at the receiving plane with vortex source under different AO working modes.

In comparison with Table 1, power efficiency of beam uplink propagation in the relay mirror system is significantly improved by using vortex source. Under the "open-loop" mode, power efficiency is improved from 49.17% to 67.45%. Under the "closed-loop" mode with turbulence completely corrected, power efficiency is improved from 86.44% to 92.53%.

3.3. Relay mirror system with vortex source and phase optimization

We calculate performance of the beam uplink propagation in the relay mirror system with the vortex source and phase optimization. Phase of the vortex source is optimized by using the stochastic parallel gradient descent algorithm. Parameters used in the optimization process are set as: samples number of phase distribution $M = 51$, the update gain is $\gamma = 0.15$, cycle index of the optimization process $m = 1000$. After calculation, we get the evaluation function on the iteration steps of the stochastic parallel gradient descent algorithm, as shown in Fig. 7, and optical intensity distribution at the receiving plane under the "closed-loop with optimization" mode,

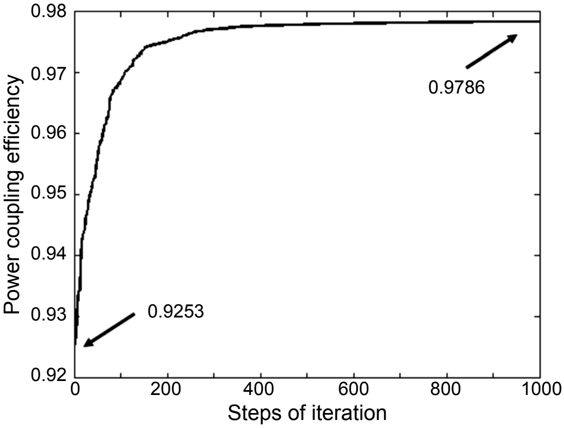

Fig. 7. The evaluation function in the optimization process.

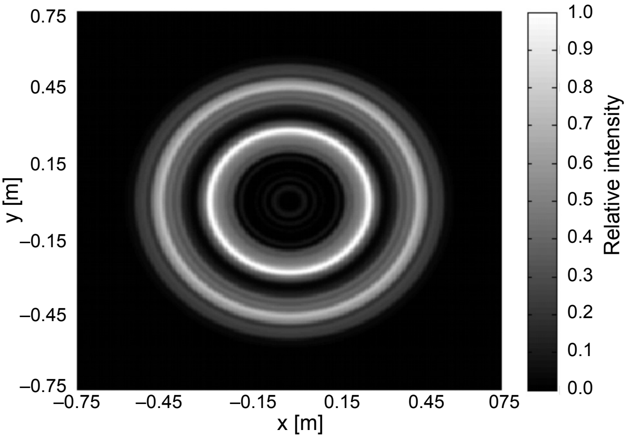

Fig. 8. Intensity distribution with vortex source and phase optimization.

Table 3. Power distribution at the receiving plane with vortex source and phase optimization.

AO mode	Power blocked by the secondary mirror	Power out of the main aperture	Power in the receiving area
Closed-loop with optimization	0.72%	1.42%	97.86%

as shown in Fig. 8. Table 3 shows power distribution at the receiving plane with the vortex source and phase optimization.

In comparison with Tab. 2, power efficiency of the beam uplink propagation in the relay mirror system with the vortex source is improved from 92.53% to 97.86% by optimizing phase distribution of the source. Power losses induced by beam blocking decreased from 0.83% to 0.72%, and power losses induced by beam truncation decreased from 6.64% to 1.42%.

4. Experimental results

Based on the above principle, we perform a reduced-scale experiment of beam uplink propagation in the relay mirror system under the "closed-loop" mode. A schematic diagram of the experiment is shown in Fig. 9.

The experimental system is composed of a source, a collimating installation, phase controllers, a detector and other units. The laser used in the experiment is an Nd:YVO_4 laser with Gaussian intensity distribution. The same two liquid crystal spatial light modulators (LC-SLM) are used in the experiment, one is used to generate the vortex beam and the other is used to control phase distribution of the vortex source. The size of the LC-SLM is 7.68 mm×7.68 mm and the LC-SLM contains

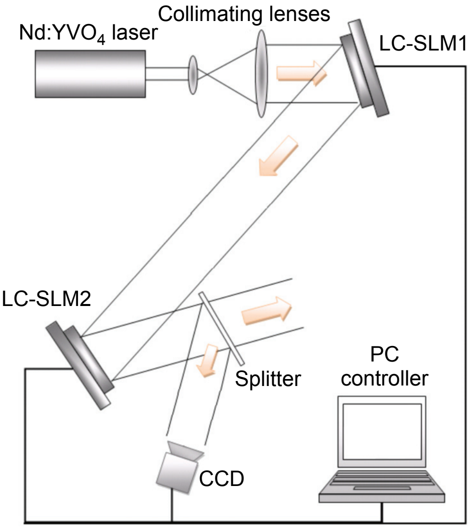

Fig. 9. Schematic diagram of the reduced-scale experimental system.

512×512 correcting units. The vortex beam is generated by using the Nd:YVO$_4$ laser and LC-SLM1 with spiral phase distribution. LC-SLM2 is used to control phase distribution of the vortex beam. A CCD camera is used as the detector and the size of a pixel is 6.45 μm×6.45 μm. The aperture of the source beam is 6 mm and the beam uplink propagation distance in the experiment is 0.75 m. The receiver is set as a telescope with 6 mm outer diameter and 1.2 mm inner diameter.

Firstly, we detect power efficiency of beam uplink propagation in the reduced-scale relay mirror system with the Gaussian source. Intensity distribution at the re-

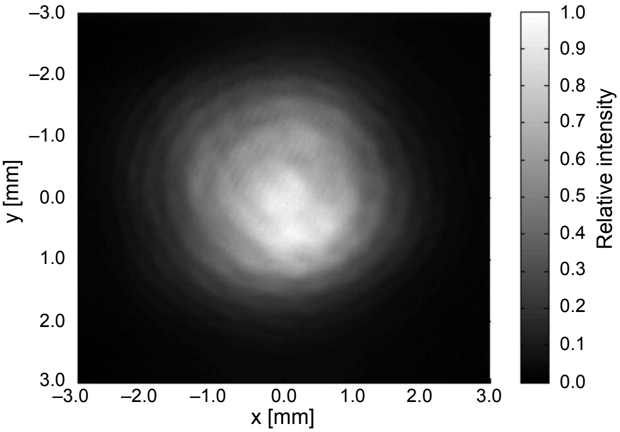

Fig. 10. Intensity distribution at the receiving plane with the Gaussian source.

ceiving plane is shown in Fig. 10. The power efficiency of beam uplink propagation is 71.89%, power losses induced by beam blocking are 18.85%, and power losses induced by beam truncation are 9.26%.

Secondly, we detect power efficiency of the beam uplink propagation in the reduced-scale relay mirror system with the Gaussian-vortex source. After calculation, we get that the optimal topological charge of the vortex beam used in the experimental system is $n = 3$. Phase distribution of the LC-SLM1 is shown in Fig. 11. In Figure 11, the coordinates denote the number of the correcting units. Intensity distribution of the receiving beam is shown in Fig.12. We get that power efficiency of beam uplink propagation is 90.60%.

Thirdly, we detect power efficiency of the beam uplink propagation in the reduced--scale relay mirror system with the Gaussian-vortex source and phase optimization. We optimized phase distribution of the source using the stochastic parallel gradient descent algorithm. Parameters used in the phase optimization process are set as:

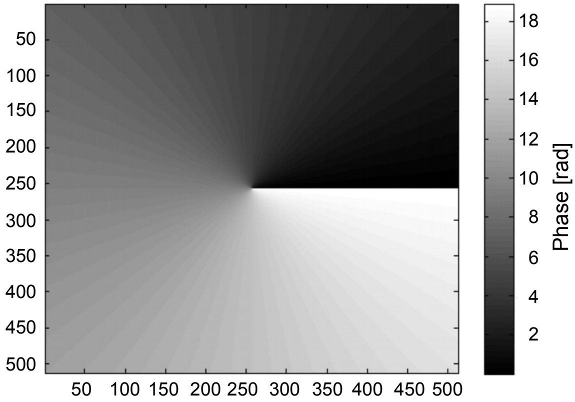

Fig. 11. Phase distribution of LC-SLM1.

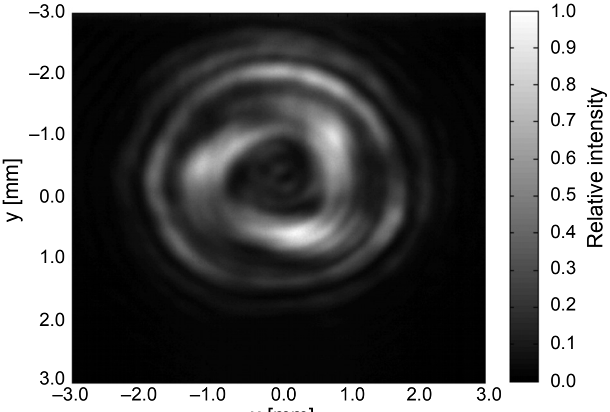

Fig. 12. Intensity distribution at the receiving plane in the system with the Gaussian-vortex source.

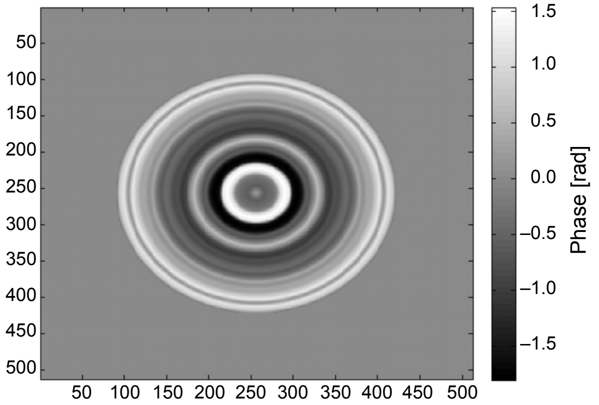

Fig. 13. Phase distribution of LC-SLM2.

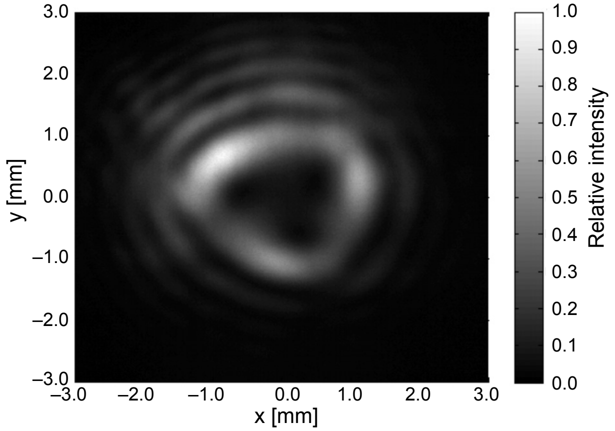

Fig. 14. Intensity distribution at the receiving plane in the system with the Gaussian-vortex source and phase optimization.

samples number of phase distribution $M = 64$, the update gain $\gamma = 0.15$, cycle index of the optimization process $m = 1000$. After calculation, the optimal phase distribution of LC-SLM2 is shown in Fig. 13. Intensity distribution of the receiving beam is shown in Fig. 14. We get that power efficiency of beam uplink propagation in the reduced-scale system is 91.59%.

5. Conclusions

In this paper, we presented a method to improve power efficiency of beam uplink propagation in the relay mirror system by using vortex source and phase optimization. A vortex beam with optimal optical topological charge is used as the system source, and phase distribution of the vortex source is optimized by using the stochastic parallel gradient descent algorithm. We analyzed power losses in the relay mirror system under different conditions, the calculation results showed that power efficiency of beam uplink propagation in the relay mirror system can be improved from 86.44% to 97.86% under the "closed-loop" mode. We performed a reduced-scale experiment and the experimental results showed that power efficiency of beam uplink propagation in the relay mirror system can be improved from 71.89% to 91.59%. In summary, power efficiency of beam uplink propagation in the relay mirror system can be effectively improved by using the vortex source and phase optimization.

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